Computation

1. If $f(1) = 4$, $f'(1) = 2$, $g(1) = 1$, and $g'(1) = -3$, find
   a. $(fg)'(1)$
   b. $(\frac{f}{f+g})'(1)$
   c. $(f \circ g)'(1)$

2. Differentiate the following. (Only minimal simplification is expected.)
   (a) $f(x) = 3x^2 - 2x + \pi$
   (b) $f(x) = x^5 e^x$
   (c) $\ln\left(\frac{\cos(x)}{x}\right)$
   (d) $f(x) = \sin x - 2 \tan x$
(e) \( f(x) = -6 \arcsin(\sqrt{x}) \)

(f) \( f(x) = \cos(\cos(\cos x)) \)

3. Find the equation for the tangent line to the function \( f(x) = \frac{1-x}{x+2} \) at \( x = 2 \).

4. Find \( \frac{dy}{dx} \) using implicit differentiation on the equation \( x^2 - 3xy + y^2 = 18 \).
5. You don’t have your calculator but you need to know the value of \( \sin(3.25) \). You do know, of course, that \( \sin(\pi) = 0 \) and that \( \cos(\pi) = -1 \). Further, you know that \( 3.25 = \pi + 0.11 \). Find the linearization (a.k.a. tangent line approximation) of \( \sin(x) \) at \( a = \pi \) and use it to estimate \( \sin(3.25) \).

6. Use linear approximation or differentials to estimate the value of:
   a. \( (1.01)^8 \)
   b. \( \frac{1}{1002} \)

7. If a rock is thrown upward with an initial velocity of 80 ft/s then the height after \( t \) seconds is \( h(t) = 80t - 16t^2 \).
   a. What is the velocity of the stone after 2 seconds?
   b. What is the velocity of the rock when it is 96 ft above the ground on the way up?
8. A particle moves on a vertical line so that its coordinate at time \( t \) is \( y = t^3 - 12t + 3, \ t \geq 0 \).
   a. Find the velocity and acceleration functions.
   b. When is the particle moving upwards and when is it moving downwards?

9. Find the points on the ellipse \( x^2 + 2y^2 = 1 \) where the tangent line has slope \( a \).

Proofs

10. Given that \( \cot x = \frac{\cos x}{\sin x} \), find, using the quotient rule, the derivative of \( \cot x \).
11. Using implicit differentiation on the equation \( x = \sin(y) \), find the differentiation formula for \( y = \arcsin(x) \).