Optimization problems are very practical. They can be used in business, transportation, medicine and much more. This is the fun thing about optimization, it is extremely applicable. Now, not all of the problems we will look at will be the most practical or interesting problems, (we would have to do a lot more work for things to get more interesting), but it is important to notice that optimization is an area of mathematics that has a variety of applications and is a very valuable area of study.

Optimization problems are about maximizing the things that you want more of (i.e. profits) and minimizing the things that you would like less of. (i.e. costs).

Luckily, in the last few sections we have discussed how to find the maximums and minimums of particular functions.

1. Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible.

2. The rate (in mg carbon/m$^3$/h) at which photosynthesis takes place for a species of phytoplankton is modeled by the function

\[ P = \frac{100l}{l^2 + l + 4} \]

where $l$ is the light intensity (measured in thousands of foot-candles). For what light intensity is the rate of photosynthesis a maximum?
3. A rectangular storage container with an open top is to have volume of 10 m$^3$. The length of its base is twice the width. Material for the base costs $10 per square meter. Material for the sides costs $6 per square meter. Find the cost of materials for the cheapest such container.

4. A boat leaves a dock at 2:00 pm and travels due south at the speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 pm. At what time were the two boats closest together?
It may be important to know the following background information about economics.

**DEFINITIONS**

- The **cost function** is the cost of producing \( x \) units of a certain product and is denoted \( C(x) \).
- The **marginal cost** is the rate of change of \( C \) with respect to \( x \), \( C'(x) \).
- The **demand (or price) function**, \( p(x) \), is the price per unit that the company can charge if it sells \( x \) units.
- The **revenue function**, \( R(x) \), is the total revenue and is modeled by \( xp(x) \) if \( x \) units are sold and the price per unit is \( p(x) \).
- The **marginal revenue function** is the rate of change of revenue with respect to the number of units sold, \( R'(x) \).
- The **profit function**, \( P(x) \) when \( x \) units are sold is modeled by \( P(x) = R(x) - C(x) \).
- The **marginal profit function**, \( P'(x) \), is the derivative of the profit function.

5. A baseball team plays in a stadium that holds 55,000 spectators. With ticket prices at $10, the average attendance has been 27,000. When ticket prices were lowered to $8, the average attendance rose to 33,000.

   a. Find the demand function, assuming that it is linear.

   b. How should ticket prices be set to maximize revenue?
6. The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is $800 per month. A market survey suggests that, on average, one additional unit will remain vacant for each $10 increase in rent. What rent should the manager change to maximize revenue?

7. During the summer months, Terry makes and sells prints on the beach. Last summer, he sold the prints for $10 each and his sales averages 20 per day. When he increased the price by $1, he found that the average decreased by 2 sales per day.

(a) Find the demand function, assuming that it is linear.

(b) If the material for each print costs Terry $6, what should the selling price be to maximize his profit?