NAME

Integration by Parts

The goal of this section is to be able to integrate more complicated functions.

We have discussed that our Substitution Rule corresponds to the Chain Rule for differentiation. We will also need a rule that corresponds to the Product Rule for differentiation.

Recall that the Product Rule states that if \( f \) and \( g \) are differentiable functions, then

\[
\frac{d}{dx} \left[ f(x)g(x) \right] = f(x)g'(x) + g(x)f'(x).
\]

In the notation for indefinite integrals, this becomes

\[
\int f(x)g'(x) \, dx = f(x)g(x) - \int g(x)f'(x) \, dx.
\]

Splitting up the integral, we see

\[
\int f(x)g'(x) \, dx = f(x)g(x) - \int g(x)f'(x) \, dx
\]

We can rearrange this equation and we get

\[
\int f(x)g'(x) \, dx = f(x)g(x) - \int g(x)f'(x) \, dx
\]

This provides us with the formula for Integration by Parts,

\[
\text{INTEGRATION BY PARTS}
\]

Let \( u = f(x) \) and \( v = g(x) \). Then the differentials \( du = f'(x) \, dx \) and \( dv = g'(x) \, dx \)

\[
\int u \, dv = uv - \int v \, du
\]

Examples

Evaluate the following indefinite integrals.

1. \( \int x \sin x \, dx \)
2. \( \int x e^{-x} \, dx \)

3. \( \int \ln x \, dx \)

4. \( \int p^5 \ln p \, dp \)

5. \( \int x^2 \sin(x) \, dx \)
\[
\int_a^b f(x)g'(x) \, dx = f(x)g(x) \bigg|_a^b - \int_a^b g(x)f'(x) \, dx
\]

Evaluate the following definite integrals.

1. \( \int_0^1 (x^2 + 1)e^{-x} \, dx \)

2. \( \int_1^2 \frac{\ln x}{x^2} \, dx \)

3. \( \int_0^1 \frac{y}{e^x} \, dx \)
4. $\int_{0}^{1} \tan^{-1} x \, dx$