NAME______________________________

Integrals of Symmetric Functions

**Suppose** \( f \) and \( g \) are continuous functions on \([-a, a]\).

(a) If \( f \) is even \([f(-x) = f(x)]\), then \( \int_{-a}^{a} f(x) \, dx = 2 \int_{0}^{a} f(x) \, dx \)

(b) If \( f \) is odd \([f(-x) = -f(x)]\), then \( \int_{-a}^{a} f(x) \, dx = 0 \)

Sketch a graph of an even function and explain why (a) is true.

Sketch a graph of an odd function and explain why (b) is true.

**proof:** Assume \( f \) is ______________________ on \([-a, a]\).

Consider \( \int_{-a}^{a} f(x) \, dx \).

We can split this integral up into the sum of two integrals as follows,

\[
\int_{-a}^{a} f(x) \, dx = \int_{-a}^{0} f(x) \, dx + \int_{0}^{a} f(x) \, dx
\]

by Property # _____ of Integrals

\[
= \text{________________________} + \int_{0}^{a} f(x) \, dx
\]

by Property # I of Integrals

(in class notes or p. 350)
For the integral, \(-\int_{-a}^{0} f(x) \, dx\), make the substitution \(u = -x\).

This means:

- the differential \(du = \underline{\phantom{1}}\)
- and when \(x = -a\), \(u = \underline{\phantom{1}}\)

In which case,

\[-\int_{0}^{-a} f(x) \, dx = \underline{\phantom{1}}\]

Furthermore, from our original equation, we get

\[
\int_{-a}^{a} f(x) \, dx = \int_{0}^{a} f(-u) \, du + \int_{0}^{a} f(x) \, dx
\]

Now, let’s consider our two different cases:

(a) Assume \(f\) is even.

This means \(f(-u) = \underline{\phantom{1}}\) and our boxed equation gives us,

\[
\int_{-a}^{a} f(x) \, dx = \underline{\phantom{1}} = \underline{\phantom{1}}
\]

(b) Assume \(f\) is odd.

This means \(f(-u) = \underline{\phantom{1}}\) and our boxed equation gives us,

\[
\int_{-a}^{a} f(x) \, dx = \underline{\phantom{1}} = \underline{\phantom{1}} = \underline{\phantom{1}}
\]