1 CHAPTER 1

2 CHAPTER 2

List and describe all of the different types of real numbers.

2.1 Counting

State and give an example of the Pigeonhole Principle.

Prove that all of the natural numbers are interesting. What principle does this proof use?

Recommended Exercises: p. 58 #1, 2, 3, 4
2.2 Patterns in Nature

List the first 8 Fibonacci numbers.

Find the non-adjacent Fibonacci numbers whose sums are: 17, 36, 54, 155.

2.3 Prime Cuts of Numbers

State the Division Algorithm.

What is the definition of a prime number?

How many primes are there?

Prove that there are infinitely many nonprime natural numbers.

What is the Prime Number Theorem and why is it interesting?

What is Fermats Last Theorem and why is it interesting?

What is the Goldbach conjecture and why is it interesting?

Recommended Exercises: p.92 # 2, 3, 7, 8, 9, 12
2.4 Crazy Clocks

What is clock arithmetic?

What mathematical concepts does modular arithmetic help us understand?

Explain how modular arithmetic relates to the Division Algorithm.

• If \( a = b \mod m \) and \( c = d \mod m \), Establish the following:
  
  A. \( a + c \equiv b + d \mod m \)

  B. \( ac \equiv bd \mod m \)

  C. \( a^2 \equiv b^2 \mod m \)

• Compute the following:
  
  A. \( 11 + 33^8 \mod 32 \)

  B. \( 10 + 145 \mod 5 \)

  C. \( 3^{930} \mod 26 \)

Recommended Exercises: p. 106 # 2, 3, 4, 5, 6, 7, 8, 12, 13, 22
2.5  PSC and How to Become a Spy

What is a Public Key Code?

Name some advantages of RSA coding.

Explain the general instructions of the RSA code (i.e. what does the receiver do and what does the sender do?)

How large do primes $p$ and $q$ have to be in the RSA coding scheme?

How does one crack an RSA code and why is this hard?

State Fermat’s Little Theorem. How does it relate to RSA?

Recommended Exercises: p. 125 # 3, 11

2.6  The Irrational Side of Numbers

Prove that the $\sqrt{2}$ is irrational.

2.7 Get Real

What does it mean for a decimal expansion to be periodic?

Given a decimal expansion can we determine if the number is rational or irrational? If so, how?

Prove that $0.9999... = 1$.

Can we give an order of the real numbers? If so, provide one.

What does it mean to say that the rational numbers are dense in the real numbers?

Practice Exercises: p.154 # 14-24