Shape and Form

In this section we will focus on some of the more aesthetic pieces of mathematics.

Chapter 12: Fractal Geometry

Here we will discuss two main examples of fractals. Fractals are geometric shapes that have a special ‘self-similarity’ property. We will examine two main examples.

The Koch Snowflake

There are two ways to think of construction the Koch snowflake:

and

There are a few things to note about the Koch snowflake:

The Sierpinski Gasket

In order to construct the Sierpinski gasket we again begin again with a rectangle; however, this time we remove triangles instead of adding them:
Chapter 13: Fibonacci Numbers and the Golden Ratio

This chapter surrounds Fibonacci’s numbers which originated from a problem known as the **Rabbit-Breeding Problem**:

> A certain man put a pair of rabbits in a place surrounded in all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair, which from the second month on become productive?

The numbers that give the general solution to this problem are called the **Fibonacci numbers**.

It turns out that the rabbit-breeding problem from Leonardo Fibonacci’s *Liber Abaci* has many applications to all sorts of growth phenomena. The play an important role in economics and computing and were even used by Yuri Matiyasevich to solve Hilbert’s 10th Problem.

**Fibonacci Numbers**

Fibonacci’s description of the problem can be restated into the following three facts:
How many pairs of rabbits are there at the end of...

Start:

Month 1:

Month 2:

Month 3:

Month 4:

Month 5:

...

Month n:

This list of integers is known as the **Fibonacci sequence**:

given by the recursive formula:

<table>
<thead>
<tr>
<th>Binet’s Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>The $n^{th}$ Fibonacci number is given by</td>
</tr>
</tbody>
</table>

or

**Example:** Find the 100$^{th}$ term in the Fibonacci sequence.
The Golden Ratio

Say you are a painter and you want to divided the plane of your canvas into two sections. You do not want the figure that you are dividing them with to be centered so the question becomes

*What kind of split would make for the ideal proportion?*

The ancient Greeks came up with a very clever answer to this question:

This is called the **divine proportion**. Notice that if we call the length of the bigger piece \( B \) and the length of the smaller piece \( S \), the divine proportion is satisfied when:

or equivalently

Doing a little manipulation...

The ratio \( \phi = \) is called the **Golden Ratio** and it represents the ratio of \( \frac{B}{S} \).
But what does this have to do with the *Fibonacci numbers* we discussed earlier? Any ideas?