The Midterm Exam will be extremely similar to these types of problems (but much shorter). I have listed
the sections from the text where you can find similar problems for extra practice and assistance.

Fully justify your answers. Among other things, this means write down all the steps you take on the
way to your answer. You are allowed a scientific calculator (or lower) but no graphing calculator, tablet,
computer, phone or any other electronic device.

From Appendix B

1. Simplify the following completely.
   (a) \((a^3b)^3(a^2b^4)^{-1}\)

   (b) \(\frac{b^{p+2q}}{(b^2)^q}\)

   (c) \(\left(\frac{2a^4b^{-1}}{4a^{-3}b^{11}}\right)^2\)

   (d) \(\sqrt[3]{16a^{12}b^2/c^5}\)

2. Factor the following completely.
   (a) \(u^2v^2 - 225\)

   (b) \(x^4 - 2x^3 + 3x - 6\)

   (c) \(t^4 - 2t^2 - 1\)

   (d) \(2x^{-1/2} + x^{3/2}\)

   (e) \((x - 3)^{1/2} - 1/3(x - 3)^{3/2}\)
Short Answer

3. State the following definitions and provide two examples (one that fits the definition and one that does not):

   (a) **function**

   (b) **inverse functions**

   (c) **one-to-one function**

4. State the following theorems:

   (a) **The Factor Theorem**

   (b) **The Fundamental Theorem of Algebra**
5. Provide two formulas for the **average rate of change** of a function and explain how the average rate of change related to the areas of increasing and decreasing for a function.

6. For two function, \( f \) and \( g \), does \( f \circ g = g \circ f \)? If so, explain why. If not, given an example of two functions \( f \) and \( g \) where this is not true.

7. What is the one necessary condition for a function’s inverse to also be a function?

**Computation**

8. (3.3) Compute the average rate of change for the function \( g(x) = -x^2 + 3x - 1 \) on the interval \([-1, 2]\).
9. (4.1) Consider \( f(x) = -3x + 7 \).

(a) Find \( f^{-1}(x) \).

(b) Find an equation for the line that is perpendicular to \( f(x) \) that passes through the point \( (4, -1) \).

(c) Find an equation for the line that passes through the points \( (0, 3) \) and \( (2, -1) \).

10. (3.4) Sketch a graph of the following functions and describe their domain and range.

(a) \( m(x) = (x + 2)^3 - 5 \)

(b) \( k(x) = \sqrt{-x} \)
(c) \( h(x) = -2(x - 1)^4 \)

(d) \( g(x) = 9 + \sqrt{8x} \)

(e) \( r(x) = \begin{cases} 2x - 1, & \text{if } x < -1 \\ x^{1/2}, & \text{if } x \geq -1 \end{cases} \)

(f) \( f(x) = x(x - 3)^2(2x + 1) \)
11. (4.2) For each function,

(i) Specify the vertex.

(ii) Specify the $x$-intercept(s) (if there are any).

(iii) Specify the $y$-intercept.

(iv) Sketch a graph of the function.

a) $f(x) = x^2 - 6x$

b) $g(x) = 2 + 9x - 3x^2$

c) $h(x) = 2x^2 - x + 14$
12. (4.7) For each function,

(i) State the domain.
(ii) Identify all vertical asymptotes.
(iii) Identify all horizontal asymptotes.
(iv) Sketch a graph of the function.

a) \( g(x) = \frac{x}{x^2 - 9} \)

b) \( h(x) = \frac{2x^2 + 2x - 12}{-x^2 - 4x + 12} \)

c) \( k(x) = \frac{4x^4 - 16}{x - 2} \)
13. (12.8) Determine the partial fraction decomposition for each of the following.

(a) \( \frac{x+18}{x^2-36} \)

(b) \( \frac{2x+1}{x^3-5x} \)

(c) \( \frac{x^3+2}{x^4-8x^2+16} \)

14. (12.2) Divide the following:

(a) \( \frac{4x^2+3x-1}{x-1} \)

(b) \( \frac{5x^4-3x^2+2}{x^2-3x+5} \)
15. (12.2) Consider the function \( f(x) = 3x^3 - 5x^2 - 16x + 12 \). Verify that \( x = -2 \) is a root and use it to factor \( f(x) \).

Word Problems

16. (4.5) What is the largest possible area for a rectangle with a perimeter of 80 cm.

17. (4.5) Suppose that the height of a ball shot straight up is given by \( h(t) = 64t - 16t^2 \) (\( h \) is measured in feet and \( t \) is measured in seconds).
   
   (a) Find the maximum height that the ball reached.

(b) Find the time at which the ball hit the ground.
18. (4.5) Find the point on the curve \( y = \sqrt{x} \) that is nearest to the point \((3, 0)\).

19. (4.5) Let \( A = 3x^2 + 4x - 5 \) and \( B = x^2 - 4x - 1 \). Find the minimum value of \( A - B \).

Proofs

20. (3.3) Let \( f(x) = ax^2 + bx + c \). Show that \( \frac{f(x+h)-f(x)}{h} = 2ax + ah + b \).
21. (4.1) Let $f$ be a linear function such that $f(a + b) = f(a) + f(b)$ for all real numbers $a$ and $b$. Show that the graph of $f$ passes through the origin.

22. (4.2) Consider $f(x) = ax^2 + bx + c$.

(a) What does the quadratic formula represent for $f(x)$?

(b) Use the method of the completing the square to establish the quadratic formula.

23. (12.3) Let $r_1$ and $r_2$ be the roots of the equation $x^2 + bx + c = 0$. Show that $r_1 r_2 = c$ and $r_1 + r_2 = -b$. 
