The purpose of this project is to familiarize you with limits. These play a large role in calculus, so this project will be helpful for those of you planning to go on to take more mathematics here at Metro State.

**Limits – Outline for Your Project**

**Step 1: Limits of Functions**

- Examine the Wikipedia entry on limits. There are two notions here that will be important to us: the limit of a function, and the limit of a sequence. You can ignore everything else on the page.
- The first paragraph of the section on ‘limits of functions’ is important for this project. Your first task is to understand it, then to restate it in your own words. (*Hint*: There should be an $\epsilon$ and a $\delta$ in there somewhere!)
- Suppose $f$ is a function. Explain why it does *not* make sense to say “the limit of the function $f$ is $L$,” but it *does* make sense to ask “what is the limit of $f(x)$ as $x$ approaches $c$?”

**Step 2: Limits of Sequences**

- Examine the wikipedia entry on ‘limits of sequences.’ There is a link to this page at the beginning of the section on limits of sequences in the article on limits. We will only be concerned with the part of this article that deals with real numbers.
- Throughout the article, the notation $(x_n)$ is used for a sequence. This is shorthand for an infinite sequence of numbers $x_0, x_1, \ldots, x_n, \ldots$. Often, rather than list the numbers, we give a formula in $n$ that tells us what the $n$th number in the sequence is.
- There is a definition near the top of the section on real numbers. Understand and restate this definition in your own words. Ensure that your rephrasing is equivalent to the original statement.
- Examine the examples.
- For each of the first three examples, write out the first five or so terms of the sequences being discussed. For example, suppose $x_n = \frac{n-3}{n+1}$. Then the first few terms in this sequence are $-3, -1, \frac{-1}{3}, 0, \frac{1}{5}, \ldots$. In your examination, make sure you explain the reasoning behind the proofs provided.

**Step 3: Understanding the Connection**

- This will be the hardest part of the project.
- Prove the following theorem, using the definitions you gave for limits of functions and limits of sequences above:

$$\text{If } \lim_{x \to a} f(x) = c \text{ and } \lim_{n \to \infty} (x_n) = a, \text{ then } \lim_{n \to \infty} f(x_n) = c.$$ 

**Step 4: Demonstrate Your Understanding**

To demonstrate that you understand what you’ve done, perform *one* of the following tasks:
• Explain the part of the article on Limits of Sequences that deals with limits in Metric Spaces. To do this, you will need to provide a functional definition of a metric space and demonstrate that you understand this definition.

• Write a Haiku or three about the connection between limits of sequences and limits of functions. Again, this is not a forum for complaints.

• Try to envision a situation in which we might want to say “the almost-limit of this function (or sequence) is 3” or something like this. Come up with a theory of almost-limits based on your idea. Note that this should not be the same as saying “the limit of this function is almost 3.”