The purpose of this project is to familiarize you with power series. These play a large role in calculus (beginning in Calculus II), so this project will be helpful for those of you planning to go on to take more mathematics here at Metro State.

**POWER SERIES – OUTLINE FOR YOUR PROJECT**

**Step 1:**
- Examine the Wikipedia entry on Taylor Series.
- There are two pictures on the right at the beginning of this entry. They will give you a good intuition of the idea of a power series, if you understand them.
- For the first picture: The colorful functions are all polynomials. You should be able to make a good guess about what polynomial each color represents. Redraw this picture and label each of the polynomials (i.e. if you think the blue function is \(x^7\), in your drawing, label that graph \(y = x^7\)).
- What do you notice about the degrees of these polynomials? There should be a couple things that stick out.

**Step 2:**
- The idea of a Taylor series is to “approximate” a non-polynomial function with polynomials. Continuing with our examination of the first picture, the colorful functions (all of which are polynomial) are supposed to be approximating the function \(y = \sin x\), which is graphed in black. Explain in your own words in what way the polynomials shown in the picture are “approximating” \(\sin x\) and how it is that these “approximations” change as we increase the degree of the polynomial.
- \(\sin x\) is an odd function. This should connect to something else you’ve already said. \(\cos x\) is an even function. Use what we’ve done so far to conjecture about how to approximate \(\cos x\) using polynomials.

**Step 3:**
- Repeat the above steps for the function \(f(x) = e^x\), which is shown in the second picture. This should be harder.

**Step 4: Demonstrate Your Understanding**
To demonstrate that you understand what you’ve done, perform one of the following tasks:
- Explain why we find it helpful to think of functions like \(\sin x\) as an “infinite polynomial.”
- We’ve now seen that we can “approximate” functions like \(\sin x\) using polynomials. Do you think we can do the converse? That is, does it seem reasonable that we can similarly “approximate” any polynomial function using things that look like

\[
A_1 \sin (B_1x + C_1) + A_2 \cos (B_2x + C_2) + A_3 \sin^2 (B_3x + C_3) + A_4 \sin (B_4x + C_4) A_5 \cos (B_5x + C_5) + \ldots
\]

Explain why you think this will or won’t work.
- By drawing graphs, try to begin a power series for \(\ln x\), similar to how we did for \(\sin x\) above. Explain what goes wrong in as detailed a way as you can.