Max/Min Problems

1. The perimeter of a rectangle is $100\text{ cm}$. How would you find the maximum area allowed.

2. What number exceeds twice its square by the greatest amount?

3. Which point on the curve $y = \sqrt{x}$ is closest to the point $(1, 0)$?
Rational Functions

We have already briefly specifically discussed one rational function.

Recall the graph of the function $f(x) = 1/x$.

Notice that $f$ has two asymptotes: one *vertical* and one *horizontal*.

**Vertical Asymptotes**

The vertical asymptotes happen at the discontinuities in the domain of the rational functions. However, not every discontinuity is a vertical asymptote.

**Examples**

Locate the discontinuities in the domains of the following functions and state which ones are vertical asymptotes.

1. $g(x) = \frac{3x-2}{(x-2)(x+2)}$

2. $h(x) = \frac{x+2}{(x+2)(x-2)}$

3. $k(x) = \frac{x-3}{2x^3-18x}$
Horizontal Asymptote

We will examine the following examples to get an idea of how to tell where the vertical asymptotes lie for a particular function. Generally, we can tell if a function has a horizontal asymptote by looking at its end behavior, so let’s see what happens to the following functions as we put in really big and really small values.

Examples

Find the horizontal asymptotes of the following functions.

1. \( g(x) = \frac{3x - 2}{(x - 2)(x + 2)} \)

2. \( f(x) = \frac{3x^3 - 2x + 1}{-2x^3 + 3x^2 - 6x} \)

3. \( h(x) = \frac{4x^4}{2x} \)
There are three cases we need to consider when determining whether or not a rational function have a horizontal asymptote (and, if so, where).

1. If the degree of the numerator is less than the degree of the denominator,

2. If the degree of the numerator is the same as the degree of the denominator,

3. If the degree of the numerator is greater than the degree of the denominator,

**Division of Polynomials**

**The Division Algorithm**

Let \( p(x) \) and \( q(x) \) be polynomials and assume that \( d(x) \) is not the zero polynomial. Then there are unique polynomials \( q(x) \) and \( R(x) \) such that

\[
p(x) = d(x) \cdot q(x) + R(x)
\]

where wither \( r(x) \) is the zero polynomial or the degree of \( R(x) \) is less than the degree of \( d(x) \).

**Long Division**

We can use the same techniques of long division to divide polynomials.

**Examples**

Perform long division on the following quotients.

1. \( \frac{60}{21} \)

2. \( \frac{x^3+2x^2-2}{x-3} \)
3. \[ \frac{2x^4+3x^3-x+1}{x^3-2x+1} \]

Synthetic Division

Examples

Perform Synthetic Division of the following quotients.

1. \[ \frac{2x^3+2x^2-2}{x-3} \]

2. \[ \frac{x^3-6x+4}{x-2} \]

What does it mean if there is no remainder when we divide one polynomial by another?