Exponential Functions

Let \( b \) denote an arbitrary positive constant other than 1. The exponential function with base \( b \) is defined by the equation

\[
y = b^x.
\]

What is the domain of such a function?

For \( b > 1 \), what happens as our \( x \)-values get small?

What is the range of such a function?

Using your calculator as a tool, sketch a graph of the following functions and identify where they are increasing and decreasing:

\[
f_1(x) = (1/2)^x \hspace{2cm} g_1(x) = 2^x
\]

\[
f_2(x) = (1/3)^x \hspace{2cm} g_2(x) = 3^x
\]
Is $f(x) = b^x$ a *one-to-one* function? Why or why not?

What does this mean about its *inverse*?

For $f(x) = b^x$, find $f^{-1}(x)$.

**Logarithmic Functions**

We define the expression $\log_b x$ to mean 'the exponent to which $b$ must be raised to yield $x$'.

In other words, $\log_3 9$ says 'what power must we raise 3 to in order to get 9?'

**Examples**

Evaluate the following.

1. $\log_4 64$

2. $\frac{\log_2 8}{\log_2 4}$
Verify that \( f(x) = b^x \) and \( g(x) = \log_b x \) are inverses of one another.

Sketch a graph of \( f(x) = 10^x \) and \( g(x) = \log_{10} x \) on the same set of axis.

What is the domain of a general logarithmic function \( (g(x)) \)?

What is the range of a general logarithmic function \( (g(x)) \)?

Two extra pieces of notation: