NAME ________________________________

Lagrange Multipliers and Constrained Optimization

We have seen optimization problems where we are requires to optimize an objective function where the variables were subject to a restraint equation.

Here is our PROBLEM:

> Let $f(x,y)$ and $g(x,y)$ be functions of two variables. Find values of x and y that maximize (or minimize) the objective function $f(x,y)$ and that also satisfy the constraint equation $g(x,y)=0$.

Normally we would like to use the function $g(x,y)=0$ to write one variable as a function of the other. However, this is not always possible. (For instance if $g(x,y)=x^4=5x^3y+7x^2y^3+y^5-17=0$.)

THE METHOD OF LAGRANGE MULTIPLIERS is our way to circumvent this problem. The basic idea is to

> replace $f(x,y)$ by an auxiliary function $F(x,y,\lambda)$ defined as $F(x,y,\lambda)=f(x,y)+\lambda g(x,y)$.

The new variable λ is called the *Lagrange multiplier* and always multiplies the constraint function $g(x,y)$.

> THEOREM
>
> Suppose that, subject to the constraint $g(x,y)=0$, the function $f(x,y)$ has a relative minimum or maximum at (a,b).
>
> Then there is a value λ–say $\lambda=c$–such that the partial derivatives of $F(x,y,\lambda)$ all equal zero at $(x,y,\lambda)=(a,b,c)$.

What is this theorem telling us?

Examples

1. Maximize $36 - x^2 - y^2$ subject to the constraint $x + 7y - 25 = 0$.

2. Minimize $42x + 28y$ subject to the constraint $600 - xy = 0$ where x and y are restricted to positive values.

Double Integrals

So far we have only been discussing derivatives of functions of several variable.

Our generalization of the definite integral for a function of one variable is the double integral over the region R denoted

$$\int\int_R f(x, y)\ dx\ dy.$$

What is R?

What does the double integral *mean*?

Examples

1. $\displaystyle\int\int_{[1,2]\times[3,4]} (y - x)\ dy\ dx$

2. $\displaystyle\int_0^1 \left(\int_0^4 x\sqrt{y} + y \; dy \right) dx$

3. $\displaystyle\int_0^1 \left(\int_{\sqrt{x}}^{x+1} 2xy \; dy \right) dx$