Exponential Functions

An exponential function is a function of the form $a^x$.

What do their graphs look like?
Use your calculator to sketch a graph of the following functions:

a. $f_1(x) = 2^x$

b. $f_2(x) = (1/2)^x$

c. $f_3(x) = 5^x$

d. $f_4(x) = (1/5)^x$

What about $f_5(x) = 2^{-x}$ and $f_6(x) = (1/2)^{-x}$?

What are some characteristics that you notice about the graphs of exponential functions?

As you can see from their graphs, exponential functions are particularly useful as they are often used to model growth (say of an investment or a bacteria). Because of this we will be able to address a significant number of applications. But before we can look at these applications let’s review some of the algebraic operations that involve exponential functions.
We have already reviewed the Laws of Exponents in Chapter 0. The same rules apply when we are considering exponential functions.

**Laws of Exponents**

\begin{align*}
(1) \quad b^x b^y &= \quad (4) \quad (b^x)^y \\
(2) \quad b^{-x} &= \quad (5) \quad (ab)^x \\
(3) \quad \frac{b^x}{b^y} &= \quad (6) \quad \left(\frac{a}{b}\right)^x
\end{align*}

**Examples:**

1. Use the properties of exponents to write the following expressions in the form $2^{kx}$ for a suitable constant $k$.
   
   (a) $4^{3x/2}$
   
   (b) $(2^{-x} \cdot 2^{5x})^{1/2}$
   
   (c) $\frac{10^x}{5^x}$

2. Let $f(x) = 3^{6x}$. Determine all $x$ for which $f(x) = 27$.

3. Solve the following for $x$.
   
   (a) $2^{4-x} = 8$
   
   (b) $(3^{2x} \cdot 3^2)^4 = 3$
   
   (c) $2^x - \frac{1}{2^x} = 0$