Antidifferentiation

Recall that we introduced the idea of an *inverse* as a function that ‘undoes’ the operation of another function. We can think of *antidifferentiation* as an inverse in this way—it undoes the operation of taking a derivative.

**Definition**

Suppose that $f(x)$ is a given function and $F(x)$ is a function having $f(x)$ as its derivative, that is $F'(x) = f(x)$. We call $F(x)$ the antiderivative of $f(x)$.

Notice that one function $f$ may have many antiderivatives $F$.

For example, the antiderivative of $f(x) = 2x$ is

**Examples:** Find the antiderivative of the following functions.

1. $f(x) = x^2$

2. $g(x) = x^3 + 4$

3. $h(x) = e^{-2x}$

Another name for the antiderivative is the *indefinite integral*.

**Indefinite Integral**

Suppose that $f(x)$ is a function whose antiderivatives are $F(x) + C$. The standard way to express this is to write

$f$ is called the integral sign and $\int f(x) \, dx$ is called the indefinite integral and stands for the antiderivative of the function.
Power Rule
\[ \int x^r \, dx = \]

Exponential Rule
\[ \int e^{kx} \, dx = \]

Log Rule
\[ \int \frac{1}{x} \, dx = \]

Basic Properties of Integrals
\[ \int (f(x) + g(x)) \, dx = \]
\[ \int k \cdot f(x) \, dx = \]

Examples
1. Determine the following.
   
   (a) \[ \int \frac{x}{3} \, dx \]
   
   (b) \[ \int x \cdot x^2 \, dx \]
   
   (c) \[ \int \frac{7}{e^{2x}} \, dx \]
   
   (d) \[ \int -3e^{-x} + 2x - 7 \, dx \]
   
   (e) \[ \int 5x^{-1} - \sqrt{x} \, dx \]
2. Find all functions $f(x)$ with the following properties.

(a) $f'(x) = 2x - e^{-x}$, $f(0) = 1$

(b) $f'(x) = 8x^{1/3}$, $f(1) = 4$

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**Definite Integral**

**Indefinite Integral**

Suppose that $f$ is a continuous function on an interval $[a, b]$ whose antiderivatives are $F$ (i.e. $F'(x) = f(x)$). The **definite integral** of $f$ from $a$ to $b$ is

$$\int_{a}^{b} f(x) \, dx =$$

**Note:** Indefinite integrals are functions and definite integrals are numbers.

**Examples:** Evaluate the following:

1. $\int_{1}^{2} x \, dx$

2. $\int_{-1}^{1} e^{-t} \, dt$
Basic Properties of Definite Integrals

\[ \int_a^b f(x) + g(x) \, dx = \]

\[ \int_a^b f(x) - g(x) \, dx = \]

\[ \int_a^b k \cdot f(x) \, dx = \]

\[ \int_a^b f(x) \, dx = \]

\[ \int_a^b f(x) \, dx = \]

\[ \int_a^c f(x) \, dx + \int_c^b f(x) \, dx = \]

Examples

1. Evaluate the following integrals.

   (a) \( \int_{-1}^2 7 \left( \frac{x^3}{3} - 3x \right) \, dx \)

   (b) \( \int_2^6 \left( \frac{3x + \sqrt{x}}{4x^3} \right) \, dx \)

   (c) \( \int_0^\ln 2 \left( \frac{e^x + e^{-x}}{2} \right) \, dx \)

   (d) \( \int_0^1 (7x + 4) \, dx + \int_1^2 (7x + 5) \, dx \)
2. Given $\int_{-1}^{1} f(x) \, dx = 2$ and $\int_{-1}^{10} f(x) \, dx = 5$ find $\int_{1}^{10} f(x) \, dx$.

3. Given $f'(t) = -12t - \frac{1}{e^t}$ compute $f(3) - f(0)$.

4. Evaluate $\int_{0}^{3} f(x) \, dx$ using the following graph: