1. The following definition of \textit{concavity} appears in a calculus text:

\[ f \text{ is concave upwards on an interval } I \text{ if on the interval } I \text{ the graph of } f \text{ appears above all its tangents.} \]

Write out an abbreviation of the statements in italics, using some or all of the symbols \( f, f', I, \mathbb{R}, ||, \neg, \forall, \exists, \& , \lor, (, ). \)

2. Let \( A \) be a set of real numbers. Express each of the following properties in abbreviated form, using abbreviations \( \Rightarrow, \Leftrightarrow, \& , \lor, \exists, \forall \), other common math symbols such as \( <, +, \times, \text{ etc} \), and names \( \mathbb{N}, \mathbb{Q}, \mathbb{R} \).

Then write out the negation of the statements in abbreviated form and simplify the statement by moving the negation symbol 'inside' as far as possible.

(a) \( A \) has no smallest member.

(b) Between any two distinct members of \( A \) there is an integer.

(c) There are two distinct members of \( A \) with no member of \( A \) in between.
(d) $\pi$ is the only irrational member of $A$.

(e) If these is a positive number in $A$, then $A$ contains no rational numbers.

3. Let $p$ be a polynomial of positive degree with coefficients from $\mathbb{N}$. That is, $p$ is a function of the form

$$p(x) = a_0 + a_1x + \cdots + a_nx^n$$

where each of $a_0, \ldots, a_n$ is a non-negative integer, $a_n \neq 0$, and $n$ is a positive integer.

(a) Show that if $a_0 \neq 1$, then there is some $k \in \mathbb{N}$ such that $p(k)$ is not a prime number.

(b) So you think that conclusion of (a) still holds if $a_0 = 1$?