Sequences

As sequence is a function whose domain is of the form ______.

We write

\[ \lim_{n \to \infty} s_n = L \text{ if} \]

Examples:

For each sequence reason informally to determine whether or not it is convergent, and if convergent, to find its limit \( L \).

If your reasoning indicates that the limit should be \( L \), use the definition of a sequence to show that in fact \( \lim s_n = L \).

If your reasoning indicates that the sequence is divergent, show this by using the definition of a sequence to show that in fact \( \lim s_n = L \) or Note 1.12.

1. \( s_n = \frac{1}{n^2} \)
2. $s_n = (-1)^n$

3. $s_n = \frac{\sin n}{n}$
4. \[ s_n = \frac{2n^2 - 3n}{3n^2 + 1} \]

Limit Theorems

**Theorem 2.1**

Suppose \( \lim s_n = L_1 \), \( \lim t_n = L_2 \) and \( c \in \mathbb{R} \), then

(a) \( \lim(s_n + t_n) = L_1 + L_2 \);
(b) \( \lim cs_n = cL_1 \)
(c) \( \lim s_n t_n = L_1 L_2 \);
(d) \( \lim \frac{s_n}{t_n} = \frac{L_1}{L_2} \) provided \( L_2 \neq 0 \) and \( \forall n \{t_n \neq 0\} \).

Examples:

1. Use Theorem 2.1 to show that \( \frac{2n^3 + 5n^2 - 7n - 13}{5n^3 + 6n + 3} \) is convergent.
2. Prove Theorem 2.1 part (b).