Line Integrals (6.1)

**Line Integrals**

A **scalar line integral** of $f$ along the $C^1$ path $\mathbf{x}$ is

$$\int_{\mathbf{x}} f \, ds =$$

A **vector line integral** of $\mathbf{F}$ along $\mathbf{x} : [a, b] \rightarrow \mathbb{R}^n$ is

$$\int_{\mathbf{x}} \mathbf{F} \cdot ds =$$

**Physical Interpretation:**

1. Calculate the scalar or vector line integral where appropriate for the following functions and paths.

   (a) $f(x, y, z) = \frac{y + z}{y + z}, \mathbf{x}(t) = (t, t, t^{3/2}), \ 1 \leq t \leq 3$

   (b) $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}, \mathbf{x}(t) = (2t + 1, t, 3t - 1), \ 0 \leq t \leq 1$
(c) \( f(x, y, z) = y - x, \ x(t) = \begin{cases} 
(2t, t) & 0 \leq t \leq 1 \\
(t + 1, 5 - 4t) & 1 < t \leq 3 
\end{cases} \)

(d) \( \mathbf{F} = y \cos z \mathbf{i} + x \sin z \mathbf{j} + x y \sin z^2 \mathbf{k}, \ x(t) = (t, t^2, t^3), \ 0 \leq t \leq 1 \)

2. (a) Find the work done by the force field \( \mathbf{F} = x^2 y \mathbf{i} + z \mathbf{j} + (2x - y) \mathbf{k} \) on a particle as the particle moves along a straight line from \((1,1,1)\) to \((2,-3,3)\).
(b) Did you solution to part (a) involve a parametrization of a curve? If so, is your answer dependent on that choice of parametrization? Why or why not?

3. Evaluate \( \int_C xy^2 \, dx - xy \, dy \) where \( C \) is the semicircular arc from (0,2) to (0,-2) traveled clockwise.