Problem 1  (10 points) Rewrite the following as partial fractions with undetermined coefficients. You only need to write down the terms with coefficients $A_1, A_2, \ldots$. For example:

\[ \frac{1}{x^2 - 1} = \frac{A_1}{x + 1} + \frac{A_2}{x - 1}. \]

You **DO NOT** need to find the value of the coefficients.

\[ \frac{466920x^4 + 31415926x^3 - 0.1234567891011x + 10^{100}}{(x^2 + x + 5)^2(x^2 + 5x + 6)(x - 1)^2} \]

1. The denominator has order 8 while the numerator has order 4. \(\Rightarrow\) The fraction is proper. 2

2. First quadratic term: \(1^2 - 4 \cdot 5 = -19 < 0 \Rightarrow\) Irreducible. 2

Second quadratic term: \(x^2 + 5x + 6 = (x + 2)(x + 3)\)

3. \[ \frac{A_1}{x - 1} + \frac{A_2}{(x - 1)^2} + \frac{A_3x + B}{(x^2 + x + 5)} + \frac{A_4x + B_4}{(x^2 + x + 5)^2} + \frac{A_6}{x + 2} + \frac{A_7}{x + 3} \]

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Problem 2  (10 points) Find the error bounds if we use the trapezoidal rule with $n = 3$ nodes to evaluate the following integral numerically:

$$\int_{0}^{1} e^{-3x} \, dx$$

**Error bound for trapezoidal rule:**

$$E \leq \frac{K \cdot (b-a)^3}{12 \cdot n^2}$$

$$(e^{-3x})' = -3e^{-3x}, \quad (e^{-3x})'' = 9e^{-3x}$$

$$\Rightarrow K = \max_{x \in [0,1]} |f''(x)| = \max_{x \in [0,1]} 9e^{-2x} = 9$$

because $e^{-x}$ is monotonically decreasing

$$E \leq \frac{9}{12} \cdot \frac{1^3}{3^2} = \frac{1}{12}.$$