The quiz is double-sided and there are 2 problems with a total score of 20. You have 20 minutes to solve all the problems on your own. Calculators are allowed but should not be useful. Please print your name and section number at the top-right corner before you start.

Problem 1 (8 points) Find

\[ \int_C \mathbf{F} \cdot d\mathbf{s} \]

where \( C \) is the line segment from \((1, 2, 0)\) to \((4, 0, -3)\) and the field is given by

\[ \mathbf{F} = \begin{bmatrix} xe^x \\ (x + z)^2 \\ xe^x \end{bmatrix} \]

\[
\mathbf{r}(t) = (1-t) \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + t \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1+3t \\ 2-2t \\ -3t \end{bmatrix}, \quad t \in [0, 1]
\]

\[ d\mathbf{s} = \frac{d\mathbf{r}}{dt} \, dt = \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix} \, dt. \]

\[ \Rightarrow \quad \int_C \mathbf{F} \cdot d\mathbf{s} = \int_0^1 \mathbf{F} \cdot \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix} \, dt 
\]

\[ = \int_0^1 -2 [(1+3t)(4+t)]^2 \, dt 
\]

\[ = \int_0^1 -2 \, dt = -2 \]


Problem 2  (12 points) Find the center of mass of three quarters of a circle in the coordinate system given in the figure. The density is assumed to be constant $\rho = 1$.

\[ C = \bar{r} = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}, \quad t \in [0, \frac{3\pi}{2}] \Rightarrow ds = \sqrt{\cos^2 t + \sin^2 t}dt = dt \]

\[ M = \int_C \rho \, ds = \int_0^{\frac{3\pi}{2}} 1 \, dt = \frac{3\pi}{2} \]

\[ \bar{x} = \frac{1}{M} \int_C x \rho \, ds = \frac{1}{M} \int_0^{\frac{3\pi}{2}} \cos t \, dt \]

\[ = \frac{1}{M} \left( \sin t \bigg|_0^{\frac{3\pi}{2}} \right) = \frac{2}{3\pi} \cdot (-1) = -\frac{2}{3\pi} \]

\[ \bar{y} = \frac{1}{M} \int_C y \rho \, ds = \frac{1}{M} \int_0^{\frac{3\pi}{2}} \sin t \, dt \]

\[ = \frac{1}{M} \left( -\cos t \bigg|_0^{\frac{3\pi}{2}} \right) = \frac{2}{3\pi} \cdot 1 = \frac{2}{3\pi} \]

CM at \((-\frac{2}{3\pi}, \frac{2}{3\pi})\)