1. Show that the vectors \((2, -3, 4, 1)^T\), \((-1, -6, 19, -11)^T\) and \((3, -2, -1, 5)^T\) are not linearly independent. (Show that the vectors are linearly dependent. Find a linear combination of these vectors that is equal to the zero vector.)

\[
\begin{bmatrix}
2 & -1 & 3 \\
-3 & -6 & -2 \\
4 & 19 & -1 \\
1 & -11 & 5
\end{bmatrix}
\begin{bmatrix}
\chi_1 \\
\chi_2 \\
\chi_3
\end{bmatrix} = 0
\]

has a nonzero solution if yes, they are linearly dependent, and vice versa.

\(\begin{bmatrix} -4 \\ 1 \\ -3 \end{bmatrix}\) is a solution \(\Rightarrow\) linearly dependent.

2. Find the general solution of the logistic equation \(\frac{dy}{dt} = (y - 2)(y - 10)\). Solve for \(y\) in the answer.

\[
\int \frac{dy}{(y-2)(y-10)} = \int dt \Rightarrow \frac{1}{8} \log \left| \frac{y-10}{y-2} \right| = t + C
\]

\[
\Rightarrow \frac{y-10}{y-2} = Ce^{8t} \Rightarrow y = \frac{10 - 2Ce^{8t}}{1 - Ce^{8t}}
\]

\(\text{Check the equation and the solution, } t \in \mathbb{R}\).

3. Given that the explicit equation of a family of curves is \(y = 1 + C(x + 4)^3\), find the differential equation for this family.

\[
\begin{align*}
y &= 1 + C(x + 4)^3 \\
y' &= 3C(x + 4)^2 \\
\text{Solve for } C. \\
C &= \frac{y-1}{(x+4)^3}
\end{align*}
\]

\(\Rightarrow\) \(y' = 3C(x + 4)^2 = 3 \frac{y-1}{(x+4)^3} (x+4)^2 = \frac{3(y-1)}{x+4}\)
1. A steel ball is heated to a temperature of 100°C and at time \( t = 0 \) is placed in water maintained at 20°C. At \( t = 5 \) minutes the temperature of the ball is 60°C. When is the temperature of the ball reduced to 30°C? Start with the differential equation.

   (1) Let \( y(t) \) be the temp of the ball.
   (2) \[ y'(t) = k(y(t) - 20) \Rightarrow (y(t) - 20)' = k(y(t) - 20) \Rightarrow y(t) - 20 = Ce^{kt} \]
   \[ \Rightarrow y(t) = 20 + Ce^{kt}, \quad t \in \mathbb{R}. \]
   (3) \( y(0) = 100 \Rightarrow C = 80. \Rightarrow y(t) = 20 + 80e^{kt} \)
   (4) \( y(5) = 60 \Rightarrow 20 + 80e^{5k} = 60 \Rightarrow k = -\frac{1}{5}\log(2) \)
   (5) \( y(t) = 30 \Rightarrow 20 + 80e^{-\frac{t}{5}} = 30 \Rightarrow 2^{-\frac{t}{5}} = \frac{1}{2} \Rightarrow t = 5 \log(2) \)

2. Show that the vectors \((1, -1, 0, 5)^T, (2, 0, -3, 4)^T, \text{ and } (-3, 2, 1, 0)^T\) are linearly independent. You need to consider a linear combination of these vectors.

   (1) Check whether
   \[
   \begin{bmatrix}
   2 & -3 \\
   0 & 2 \\
   5 & 4 \\
   \end{bmatrix}
   \begin{bmatrix}
   x_1 \\
   x_2 \\
   x_3 \\
   \end{bmatrix}
   = \begin{bmatrix}
   0 \\
   0 \\
   0 \\
   \end{bmatrix}
   \]
   \( \rightarrow \) if yes, they are linearly dependent.
   \( \rightarrow \) if no, they are linearly independent.

   (2) Solution
   \[
   \begin{bmatrix}
   x_1 \\
   x_2 \\
   x_3 \\
   \end{bmatrix}
   = \begin{bmatrix}
   0 \\
   0 \\
   0 \\
   \end{bmatrix}
   \Rightarrow \) they are linearly independent.

3. Show that the vector \((17, -11, -7, 7)^T\) is a linear combination of the vectors \((5, -2, 0, 3)^T, (1, -4, -1, 0)^T, \text{ and } (0, 3, 2, 1)^T\).

   (1) Check whether
   \[
   \begin{bmatrix}
   5 & 1 & 0 \\
   -2 & -4 & 3 \\
   0 & -1 & 2 \\
   3 & 0 & 1 \\
   \end{bmatrix}
   \begin{bmatrix}
   x_1 \\
   x_2 \\
   x_3 \\
   \end{bmatrix}
   = \begin{bmatrix}
   17 \\
   -11 \\
   -7 \\
   7 \\
   \end{bmatrix}
   \]
   \( \rightarrow \) if yes, then this is a linear combination of the columns on the left.

   (2) Solution
   \[
   \begin{bmatrix}
   x_1 \\
   x_2 \\
   x_3 \\
   \end{bmatrix}
   = \begin{bmatrix}
   4 \\
   -3 \\
   -5 \\
   \end{bmatrix}
   \Rightarrow\) So it is a linear combination of the given vectors.