You have 10 minutes for the following problems. Calculators should NOT be useful. Please print your name and section number at the top-right corner before you start.

**Problem 1** (10 points) Let $\Omega$ be the solid body bounded by $x^2 + y^2 + z^2 = 1$ and $y^2 = x^2 + z^2$. Find the volume of $\Omega$.

Clearly, we should use spherical coordinates. The zenith direction is $y$.

\[ x = \rho \sin \phi \cos \theta \]
\[ y = \rho \cos \phi \]
\[ z = \rho \sin \phi \sin \theta \]

We clearly have $\rho \in [0, 1]$, $\theta \in [0, 2\pi]$. We still need to find the range of $\phi$.

As always, we take a $y$-$r$ section:

The maximum of $\phi$ is given by the intersection of the two surfaces.

\[ y^2 = x^2 + z^2 \Rightarrow \rho^2 \cos^2 \phi = \rho^2 \sin^2 \phi \Rightarrow \tan \theta = 1 \]

\[ \theta = \frac{\pi}{4} \]

\[ \phi = \frac{\pi}{2} \]

\[ \phi = 0 \]

\[ \phi = \frac{\pi}{4} \]

Therefore, $\phi$ ranges from $0$ to $\frac{\pi}{4}$.

\[ V = \int_0^{2\pi} \int_0^{\pi/4} \int_0^1 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \]

\[ = \frac{2\pi}{3} \int_0^{\pi/4} \sin \phi \, d\phi = \frac{2\pi}{3} \left[ -\cos \phi \right]_0^{\pi/4} \]

\[ = \frac{2\pi}{3} \left( 1 - \frac{\sqrt{2}}{2} \right) \]