Questions 2,9 are not graded, so solutions are given. For graded questions, only comments are given. Questions are ordered as on the website.

**Question 1:** Need to check \( d\psi \) is surjective. Locally one has \( d\psi(A) = \lim_{h \to 0} \frac{\psi(I + hA) - \psi(I)}{h} = A + A^t \). So \( \dim \text{Im}(d\psi) = d^2 - \dim \text{Ker}(d\psi) = \frac{d(d+1)}{2} \) is surjective.

**Question 2:** (a) Locally write \( X = \sum a_i \partial x_i \) and \( Y = \sum b_i \partial x_i \), where \( a_i, b_i \) are smooth functions. Then \( [X,Y] = \sum_i \sum_j (a_i \frac{\partial b_j}{\partial x_i} - b_i \frac{\partial a_j}{\partial x_i}) \partial x_j \), which is a smooth vector field.

(b) One can check in local coordinates. Coordinate freely, one can write \( [fX,gY] = (fX)(gY) - (gY)(fX) = f((Xg)Y + gXY) - g((Yf)X + fYX) = fg[X,Y] + f(Xg)Y - g(Yf)X \).

(c)(d) Trivial computation.

**Question 3:** Easy computation.

**Question 4:** Because of compactness, we can get a lower bound \( \varepsilon \) such that locally at any point, an integral curve exists in time \((-\varepsilon, \varepsilon)\). So suppose a maximal integral curve at \( p \) is \( \gamma : (a,b) \to M \), one can extend the curve at \( \gamma(b - \varepsilon/4) \) to \( \tilde{\gamma} : (a, b + \varepsilon/2) \), contradicting the maximality.

**Question 5:** For \( f : \mathbb{R}^2 \to \mathbb{R} \), \( df \) is either identically 0 or rank 1. In the former case, \( f \) is constant map. In the latter case, \( f \) must have 1 dimensional preimage.

**Question 6:** Easy computation.

**Question 7:** Take \( \text{det} : \mathbb{R}^4 \setminus \{0\} \to \mathbb{R} \) to be the determinant. Then one can show \( \text{det} \) is submersion and \( \text{det}^{-1}(0) \) is exactly the space of rank 1 matrices.

**Question 8:** It’s easy to show it is immersion and injective. To show density, one can show \( \text{Im}(f) \cap \{x\} \times S^1 \) is dense for all \( x \). However, this is not an embedding because for any open \( U \) in \( S^1 \times S^1 \), \( \text{Im}(f) \cap U \) is not path connected. So \( f \) is not homeomorphic onto the image.
Question 9: For $v \in P_m$, let $\gamma$ be an integral curve in a neighborhood of $m$ in $P$ such that $\gamma(0) = m$ and $\dot{\gamma}(0) = v$. Then we have $d\psi(di(v)) = \frac{d}{dt} \bigg|_{t=0} (\psi(i(\gamma(t)))) = 0$ because $\psi \circ i \circ \gamma$ is a constant map.