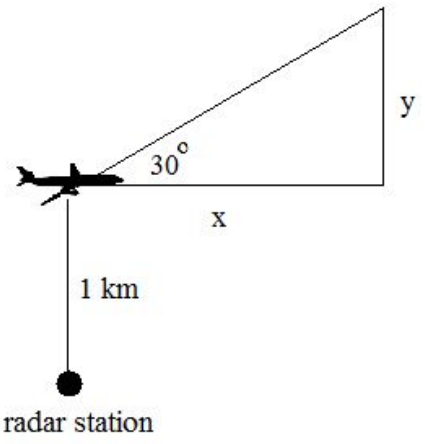
MATH 1271

Example Related Rates Problem

A plane flying with a constant speed of 300 km/hr passes over a ground radar station at an altitude of 1 km and climbs at an angle of 30°. At what rate is the distance from the plane to the radar station increasing a minute later?

First try drawing a picture of this scenario for yourself. Below is my picture:



Now we can write x and y as functions of time.

$$x = 5t\cos 30^\circ$$
$$y = 5t\sin 30^\circ$$

Now the distance $D = \sqrt{x^2 + (1+y)^2}$. And after one minute

$$D = \sqrt{\left(\frac{5\sqrt{3}}{2}\right)^2 + \left(1 + \frac{5}{2}\right)^2} = \sqrt{31}$$

But it will be easier to work with D^2.

$$\begin{aligned} D^2 &= x^2 + (1+y)^2 \\ &= (5t\cos 30^\circ)^2 + (1 + 5t\sin 30^\circ)^2 \\ &= 25t^2\cos^2(30^\circ) + 25t^2\sin^2(30^\circ) + 10t\sin(30^\circ) + 1 \\ &= 25t^2 + 10t\sin(30^\circ) + 1 \\ &= 25t^2 + 5t + 1 \end{aligned}$$

Then differentiating the above we get $2D\frac{dD}{dt} = 50t + 5$. So when $t = 1$, we get

$$\begin{aligned} \frac{dD}{dt} &= \frac{50(1)+5}{2D} \\ &= \frac{55}{2\sqrt{31}} \end{aligned}$$

So our answer is $\frac{55}{2\sqrt{31}}$ km/min which when converted to kilometers per hour becomes

$$\boxed{\frac{1650}{\sqrt{31}} \text{ km/hr}}.$$