Below is a worked out two related rates problem that I thought might be helpful.

1. A screen saver displays the outline of a 3cm by 2cm rectangle and then expands it in such a way that the 2cm side is expanding at at the rate of 4cm/sec and the proportions of the rectangle never change. How fast is the area of the rectangle increasing when its dimensions are 12cm by 8cm?

First we will draw a picture:

We have the following relations:

\[ x = 12 \]
\[ y = 8 \]
\[ \frac{dy}{dt} = 4 \]
\[ A = xy \]

We want to find the rate at which the area of the rectangle is increasing so we must differentiate the last relation with respect to time. This yields

\[ \frac{dA}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt} \] (1)

But we know all of these values except for \( \frac{dx}{dt} \). A look back at the problem statement tells us that when the rectangle increases, its proportions never change. Moreover, we have that \( x = 3 \) and \( y = 2 \) at the start so this tells us that \( 2x = 3y \). When we differentiate this equation with respect to time we get

\[ 2\frac{dx}{dt} = 3\frac{dy}{dt} \]

This tells us that \( \frac{dx}{dt} = 6 \) So we can plug in the values for each variable into (1) and get

\[ \frac{dA}{dt} = 6(8) + 12(4) \]
\[ = 96 \text{ cm}^2/\text{sec} \]