(1) Let $A$ be a set of real numbers. Express each of the following properties in abbreviated form, using the abbreviations $\Rightarrow$, $\iff$, $\&$, $\lor$, $\exists$, $\forall$, other common math symbols, and the names $N$, $Q$, and $R$.

**Note:** Your answers may vary slightly compared to mine.

(a) $A$ has no smallest member.

$$\forall x \in A \exists y \in A [y < x]$$

(b) $\pi$ is the only irrational member of $A$.

$$\forall x \in A [x = \pi \lor x \in Q]$$

(c) If there is a positive number in $A$, then $A$ contains no rational numbers.

$$\exists x \in A [x > 0] \Rightarrow (\forall x \in A [x \in Q])$$

(d) Between any two distinct members of $A$ there is an integer.

$$\forall x \in A \forall y \in A [x \neq y] \Rightarrow (\exists z \in Z [x \leq z \leq y \lor y \leq z \leq x])$$

(e) There are two distinct members of $A$ with no member of $A$ in between.

$$\exists x \in A \exists y \in A \exists z \in A [x \leq z \leq y \lor y \leq z \leq x]$$

(2) For each of the parts of the previous problem, write out the negation of the statements in abbreviated form. Then simplify the statement by moving the negation symbol 'inside' as far as possible.

(a) $\exists x \in A [\forall y \in A [x \leq y]]$

(b) $\exists x \in A [x \neq \pi \& \neg(x \in Q)]$

(c) $$(\exists x \in A [x > 0]) \& \neg(\forall x \in A [x \in Q])$$

$$\neg(\exists x \in A [x > 0] \& (\exists x \in A [x \in Q])]$$
(d) 
\[
\exists x \in A \neg \left( \forall y \in A \left( x \neq y \Rightarrow (\exists z \in \mathbb{Z} \left( x \leq z \leq y \lor y \leq z \leq x \right)) \right) \right) \\
\exists x \in A \exists y \in A \left( x \neq y \Rightarrow (\exists z \in \mathbb{Z} \left( x \leq z \leq y \lor y \leq z \leq x \right)) \right) \\
\exists x \in A \exists y \in A \left( x \neq y \& (\forall z \in \mathbb{Z} \left( x \leq z \leq y \lor y \leq z \leq x \right)) \right) \\
\exists x \in A \exists y \in A \left( x \neq y \& (\forall z \in \mathbb{Z} \left( x \leq z \leq y \lor y \leq z \leq x \right)) \right) \\
\left( \exists x \in A \exists y \in A \left( x \neq y \& (\forall z \in \mathbb{Z} \left( z < x \lor z > y \right)) \right) \right)
\]

(e) 
\[
\neg \left( \exists x \in A \exists y \in A \neg [\exists z \in A (x \leq z \leq y \lor y \leq z \leq x)] \right) \\
\forall x \in A \neg \left( \exists y \in A \neg [\exists z \in A (x \leq z \leq y \lor y \leq z \leq x)] \right) \\
\forall x \in A \forall y \in A [\exists z \in A (x \leq z \leq y \lor y \leq z \leq x)]
\]

(3) For each of the parts of problem 1, come up with an example of a set \( A \) of real numbers such that:

(i) The statement is true.
(ii) The statement is false.

Note: Your answers may vary from mine.

(a) (i) \( A = (0, 1) \) (Notice this is the open interval).
    (ii) \( A = \mathbb{N} \)
(b) (i) \( A = \{ \pi \} \)
    (ii) \( A = \mathbb{R} \)
(c) (i) \( A = \{-1, -2, -3, \pi, -10, 0\} \)
    (ii) \( A = \{1\} \)
(d) (i) \( A = \{0.5, 1.5\} \)
    (ii) \( A = \mathbb{Q} \)
(e) (i) \( A = \{1, 2\} \)
    (ii) \( A = \mathbb{R} \)
(4) Determine whether the set is finite or infinite

(a) the empty set. finite
(b) The set of real numbers between 1 and 100 (inclusive). infinite
(c) The set of integers between 1 and 100 (inclusive). finite
(d) The set of integers between 1 and 100 (exclusive). finite
(e) The set of students that attend the University of Minnesota in the year 2016. finite
(f) A subset of a finite set. finite
(g) Each set you came up with in problem 3.