(1) Prove (not necessarily using the definition of limit) the limits of the following sequences:

(a) \( a_n = e^{-n} \sin(n) \)

(b) Suppose \( x > 0 \). \( a_n = \cos(\pi x^{1/n}) \)

(2) (This problem courtesy of one of your classmates): If \( a_n \) is a bounded (not necessarily convergent) sequence and \( b_n \to 0 \), then does \( \{a_n b_n\} \) converge or diverge? If it converges, find its limit. In either case, prove it.
(3) Let

\[ f = \begin{cases} 
  1, & \text{if } x > 0; \\
  0, & \text{if } x \leq 0, 
\end{cases} \]

and

\[ s_n = \frac{1}{n} \]

(a) Write out the first few terms of \( \{f(s_n)\} \)
(b) What is \( \lim f(s_n) \)?
(c) What is \( \lim s_n \)? What is \( f(\lim s_n) \)?
(d) Why doesn’t this contradict our theorem about sequences and continuous functions?