(1) Prove that the sequence defined by $a_1 = 1$, $a_{n+1} = \frac{1}{3}(a_n + 4)$ converges. Find the limit.

(2) Let $p > 0$. Prove that the sequence defined by $x_1 = \sqrt{p}$, $x_{n+1} = \sqrt{p + x_n}$ is convergent and find what it converges to.

[Hint: First show that sequence is monotone (increasing or decreasing?). Then show the sequence is bounded above by $1 + 2\sqrt{p}$.]
(3) Prove that the sequence defined by \( a_1 = \frac{3}{2}, \ a_{n+1} = \frac{3}{4 - x_n} \) converges. Find the limit.

(4) For which values of (nonzero) \( c \) does the following recursively defined sequence converge or diverge?

\[
\begin{align*}
&\begin{cases}
  s_1 = c \\
  s_{n+1} = s_n + \frac{1}{s_n}
\end{cases}
\end{align*}
\]