Determine whether the improper integral is convergent or divergent. If convergent, try to find its value. It is not enough to simply give the answer. You should also show how you arrived at your answer.

(1) Determine whether the infinite series below is convergent or divergent. If it is convergent, find its value.

\[ \frac{4}{5} - \frac{8}{25} + \cdots + \frac{(-2)^{n+1}}{5^n} + \cdots \]

(2) Determine whether the statement is true or false. If true, give a proof. If false, give a counterexample.

(a) If \( \sum_{n=1}^{\infty} (a_n + b_n) \) is convergent, then \( \sum_{n=1}^{\infty} a_n \) is convergent and \( \sum_{n=1}^{\infty} b_n \) is convergent.

(b) If \( \sum_{n=1}^{\infty} (a_n b_n) \) is convergent, then \( \sum_{n=1}^{\infty} a_n \) is convergent and \( \sum_{n=1}^{\infty} b_n \) is convergent.

(c) If \( \sum_{n=1}^{\infty} a_n \) is divergent and \( \sum_{n=1}^{\infty} b_n \) is divergent, then \( \sum_{n=1}^{\infty} (a_n + b_n) \) is divergent.
(d) If $\sum_{n=1}^{\infty} a_n$ is convergent and $\sum_{n=1}^{\infty} (a_n + b_n)$ is convergent, then $\sum_{n=1}^{\infty} b_n$ is convergent.

(e) If $\sum_{n=1}^{\infty} a_n$ is convergent and $\sum_{n=1}^{\infty} (a_n b_n)$ is convergent, then $\sum_{n=1}^{\infty} b_n$ is convergent.

(3) Suppose Chicago and the Twin Cities are exactly 400 miles apart with a straight railroad track joining the two cities. Tran A, traveling at 40 mph, leaves Chicago headed for the Twin Cities. At the same time train B, also traveling at 40 mph, leaves the Twin Cities headed for Chicago, on the same track. At the time of departure, a Minnesota mosquito sitting on the engine of train B leaves for Chicago flying at 60 mph. When it meets train A it turns around and heads back towards train B, and upon meeting train B it turns around again, etc. It continues doing this as long as possible.

(a) Calculate the time $t_1$ it takes for the mosquito to travel from Train A to Train B the first time. Then calculate the distance $d_1$ the mosquito traveled.

(b) Calculate the time $t_2$ it takes for the mosquito to travel from Train B to Train A (the second leg of the mosquito’s journey). Then calculate $d_2$, the distance.

(c) Find the pattern of the above (the time/distance is geometric) and write the distance the mosquito travels as an infinite sum.

(d) Check your work by finding how long it takes the trains to collide and multiply this time by the speed of the mosquito.

(e) Ask your TA for an interesting anecdote involving the above problem.