(1) Prove that series \( \sum_{n=1}^{\infty} \frac{(-1)^{n+3} \sqrt{n}}{n + 4} \) converges and find out how many terms must be included in the partial sum for the error to be less than 0.1.

We will use the Alternating Series Test. First we must ensure that the series satisfies the hypotheses of the test. First it is obvious that \( \frac{\sqrt{n}}{n + 4} > 0 \) for all \( n \). Now we check that the sequence is eventually decreasing:

\[
\frac{d}{dx} \frac{\sqrt{x}}{x + 4} = \frac{x + 4 - \sqrt{x}}{(x + 4)^2} = \frac{\sqrt{x} + \frac{x}{2} - \sqrt{x}}{(x + 4)^2} < 0 \text{ for all } x \geq 9
\]

Finally, we check that the limit of the terms is 0:

\[
\lim_{n \to \infty} \frac{\sqrt{n}}{n + 4} \cdot \frac{1}{\sqrt{n}} = \lim_{n \to \infty} \frac{\sqrt{1/n}}{1 + 4/n} = 0
\]

So we conclude that \( \sum_{n=1}^{\infty} \frac{(-1)^{n+3} \sqrt{n}}{n + 4} \) converges by the Alternating Series Test.

Now we want to estimate the sum. We are looking for \( n - 1 \) such that \( \frac{\sqrt{n}}{n + 4} < 0.1 \). We see \( \sqrt{91} > 0.1 \) but \( \sqrt{92} < 0.1 \). Therefore the value we are looking for is:

\[
\sum_{n=1}^{91} \frac{(-1)^{n+3} \sqrt{n}}{n + 4}
\]

(2) Determine the radius of convergence and interval of convergence for the following power series:

\[
\sum_{n=1}^{\infty} \frac{2^n}{n} (4x - 8)^n
\]

We use the Ratio Test on \( \frac{2^n}{n} (4x - 8)^n > 0 \):

\[
\lim_{n \to \infty} \left| \frac{2^{n+1}(4x - 8)^{n+1}}{n + 1} \cdot \frac{n}{2^n(4x - 8)^n} \right| = \lim_{n \to \infty} \left| \frac{2(4x - 8)}{n + 1} \right| = |8(x - 2)| = 8|x - 2|
\]

So the power series will converge if \( 8|x - 2| < 1 \iff |x - 2| < \frac{1}{8} \iff \frac{15}{8} < x < \frac{17}{8} \) and will diverge if \( 8|x - 2| > 1 \iff x > \frac{17}{8} \) or \( x < \frac{15}{8} \). So the radius of convergence is \( \frac{1}{8} \).

To find the interval of convergence we need to check the endpoints: First when \( x = \frac{15}{8} \):

\[
\sum_{n=1}^{\infty} \frac{2^n}{n} (-1/2)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}
\]
which converges as it is the alternating harmonic series.
When \( x = \frac{17}{8} \):

\[
\sum_{n=1}^{\infty} \frac{2^n}{n} \left(\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n}
\]

which diverges as it is the harmonic series.

So the interval of convergence is \( \left\[ \frac{15}{8}, \frac{17}{8} \right) \).

(3) Suppose that the power series \( \sum_{n=0}^{\infty} a_n x^n \) is convergent at \( x = -3 \) and divergent at \( x = 5 \).

What can be said about

(a) convergence at \( x = -2 \)?
(b) absolute convergence at \( x = 2 \)?
(c) convergence at \( x = -6 \)?
(d) divergence at \( x = -5 \)?

The power series is centered about 0. So we know that at the very least the power series will converge on \([-3, 3]\) and will definitely diverge on \((-\infty, -5) \cup [5, \infty)\). But we don’t know about convergence or divergence on \([-5, -3) \cup [3, 5)\). So

(a) The power series **converges.**
(b) The power series **absolutely converges.**
(c) The power series **diverges.**
(d) **Nothing.**

(4) Find a power series whose interval of convergence is

(a) \([2, 8]\).
(b) \((2, 8)\).
(c) \((2, 8]\).
(d) \([2, 8)\).

(a) \( \sum_{n=1}^{\infty} \frac{1}{3^n n^2} (x - 5)^n \)
(b) \( \sum_{n=1}^{\infty} \frac{n}{3^n} (x - 5)^n \)
(c) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} (x - 5)^n \)
(d) \( \sum_{n=1}^{\infty} \frac{1}{n^3} (x - 5)^n \)