(1) Find the maximum and minimum values of the function $f(x, y) = 4x + 6y$ on the closed disk centered about the origin with radius $\sqrt{13}$.

minimum = $-26$  
maximum = $26$

(2) $\int_1^3 \int_0^1 (1 + 4xy)\,dx\,dy = 10$

(3) $\int_1^4 \int_0^1 (2x + y)^3\,dx\,dy = \frac{1}{180}(6^{10} - 4^{10} - 3^{10} + 1)$

(4) $\int_{\ln 2}^{\ln 5} \int_{\ln 2}^{\ln 5} e^{2x-y} \,dx\,dy$

\[
\int_{\ln 2}^{\ln 5} \int_{\ln 2}^{\ln 5} e^{2x-y} \,dx\,dy = \int_{\ln 2}^{\ln 5} \left[ \frac{1}{2} e^{2x - y} \right]_{x=0}^{x=\ln 5} dy
\]
\[
= \frac{1}{2} \int_{\ln 2}^{\ln 5} (e^{2\ln 5} - e^{-y}) dy
\]
\[
= \frac{1}{2} \int_{\ln 2}^{\ln 5} (e^{2\ln 5} e^{-y} - e^{-y}) dy
\]
\[
= \frac{1}{2} \int_{\ln 2}^{\ln 5} e^{2\ln 5 - y} - e^{-y} dy
\]
\[
= \frac{1}{2} \int_{\ln 2}^{\ln 5} (25e^{-y} - e^{-y}) dy
\]
\[
= \frac{1}{2} \int_{\ln 2}^{\ln 5} 24e^{-y} dy
\]
\[
= \int_{\ln 2}^{\ln 5} 12e^{-y} dy
\]
\[
= \left[ -12e^{-y} \right]_{y=\ln 2}^{y=\ln 5}
\]
\[
= -12e^{-\ln 2} + 12
\]
\[
= -12e^{-\ln(1/2)} + 12
\]
\[
= -12 \left( \frac{1}{2} \right) + 12
\]
\[
= 6
\]

(5) $\int_R \cos(x + 2y)\,dA, \quad R = \{(x, y) | 0 \leq x \leq \pi, 0 \leq y \leq \pi/2\}$

$-2$
(6) \[ \int \int_R xy e^{x^2y} \, dA, \quad R = [0, 1] \times [0, 2] \]

To illustrate that a problem can become easier by flipping the order of integration, we will attempt to solve the problem in two ways.

\[ \int \int_R xy e^{x^2y} \, dA = \int_0^1 \int_0^2 xy e^{x^2y} \, dy \, dx \]

Notice that this way involves integration by parts. I don’t feel like doing that so let’s see what happens when we flip the limits of integration:

\[ \int \int_R xy e^{x^2y} \, dA = \int_0^2 \int_0^1 xy e^{x^2y} \, dx \, dy \]

\[ = \int_0^2 \left[ \frac{e^{x^2y}}{2} \right]_{x=0}^{x=1} \, dy \]

\[ = \frac{1}{2} \int_0^2 (e^y - 1) \, dy \]

\[ = \frac{1}{2} \left[ e^y - y \right]_{y=0}^{y=2} \]

\[ = \frac{1}{2} \left[ e^2 - 2 - (1 - 0) \right] \]

\[ = \frac{e^2 - 3}{2} \]

Wow! That way was much easier. What a great problem!

(7) Find the volume of the solid that lies under the plane \( 3x + 2y + z = 12 \) and above the rectangle \( R = \{(x, y) | 0 \leq x \leq 1, -2 \leq y \leq 3\} \)

Notice that we can rewrite the equation of the plane as \( z = 12 - 3x - 2y \). So define the function \( f(x, y) = 12 - 3x - 2y \). It is easy to see that \( f \) is positive on \( R \). So the volume of the solid that lies under the plane is equal to the double integral of \( f \) over \( R \). That is

\[ V = \int \int_R f(x, y) = \frac{95}{2} \]