(1) Let $D^*$ be the parallelogram with vertices at $(2, 4), (0, 0), (6, 2)$, and $(8, 6)$, and let $D$ be the unit square: $D = [0, 1] \times [0, 1]$. Find a $T$ such that $D$ is the image set of $D^*$ under $T$.

$$T(x, y) = \left( \frac{x}{5} - \frac{y}{10}, -\frac{3y}{10} - \frac{1}{10} \right)$$

There are other possible linear transformations that will work as well, but I think this one is the simplest to come up with.

(2) Determine if the following functions $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ are one-to-one and/or onto.

(a) $T(x, y) = (x + 2, |y|)$ \textbf{neither}

(b) $T(x, y) = (x^2 + y, y^3)$ \textbf{one-to-one and onto}

(c) $T(u, v) = (4v + 3, u)$ \textbf{neither}
(3) In front of you are three boxes. One contains only apples, one contains only oranges and one contains a mix of apples and oranges. Each box is labeled, like this:

- Box 1: apples
- Box 2: oranges
- Box 3: apples & oranges

Unfortunately, all three boxes are mislabeled. That's where you come in, you're going to help me fix those labels.

Here’s the challenge: You get to choose one, and only one, box. I will remove a randomly selected piece of fruit from your chosen box and show it to you (so you can tell if it's an apple or an orange). After that, you will be able to accurately and definitively relabel all three boxes.

(4) What is the smallest three-digit palindrome divisible by 18? [A palindrome is a number or word that is the same backwards as it is forwards. Like “racecar” or “go hang a salami I’m a lasagna hog.”]

(5) You need to enter a castle that is surrounded by a five-meter wide rectangular moat filled with water. You really don’t want to get wet and only have two 4.8-meter-long planks with no rope, nails, etc. How do you get across?

(6) You are blindfolded, and are told if you can correctly solve the following, the blindfold will be removed. You are given 99 coins that are heads up, and an unknown number of coins that are tails up. You never remove the blindfold, you do not peek. You can count the coins, put them in arbitrary many piles, flip whichever coins you want, but remember, when you flip and when you sort, you DO NOT know which ones are heads up, which are tails up. In the end, you must end up with just two piles, each containing an equal number of heads. How do you do this?

(7) An executioner lines up 100 prisoners single file and puts a red or a black hat on each prisoner’s head. Every prisoner can see the hats of the people in front of him in the line. S/he cannot see the his/her own hat or the hats of the people behind him/her in the line. If the prisoner answers correctly, s/he is allowed to live. If the prisoner gives the wrong answer, the prisoner is killed. On the night before the line-up, the prisoners confer on the strategy to help them. What is the optimal strategy? How many prisoners are guaranteed to live?