(1) Find the curl and the divergence the vector fields:

(a) \((\ln(x), yz, \sin(x^2z))\)

\[
\begin{align*}
\text{divergence} &= \frac{1}{x} + z + x^2 \cos(x^2z) \\
\text{curl} &= (-y, -2xz \cos(x^2z), 0)
\end{align*}
\]

(b) \(e^{xy}j + y \arctan(x/z)k\)

\[
\begin{align*}
\text{divergence} &= xe^{xy} + \frac{-y}{z^2 + x^2} \\
\text{curl} &= \left(\arctan(x/z), -\frac{yz}{x^2 + z^2}, ye^{xy}\right)
\end{align*}
\]

(2) Prove that for any vector field \(\mathbf{F}\), \(\text{div} \ \text{curl} \ \mathbf{F} = 0\)

Simply let \(\mathbf{F} = (F_1, F_2, F_3)\) and then calculate \(\text{div} \ \text{curl} \ \mathbf{F}\).

(3) Determine whether or not the vector field is conservative. If it is conservative, find a function \(f\) such that \(\mathbf{F} = \nabla f\).

(a) \(\mathbf{F}(x, y, z) = (y^2z^3, 2xyz^3, 3xy^2z^2)\)

\[
f(x, y, z) = xy^2z^3
\]

(b) \(\mathbf{F}(x, y, z) = 2xyi + (x^2 + 2yz)j + y^2k\)

\[
f(x, y, z) = x^2y + y^2z
\]

(c) \(\mathbf{F}(x, y, z) = (ye^{-x}, e^{-x}, 2z)\)

Not conservative

(d) \(\mathbf{F}(x, y, z) = e^zi + j + xe^zk\)

\[
f(x, y, z) = xe^z + y
\]

(4) Evaluate \(\int_C \mathbf{F} \cdot \mathbf{ds}\) where \(\mathbf{F} = \left(\frac{y^2}{1 + x^2}, 2y \arctan x\right)\) and \(C\) is the curve \(\mathbf{r} = t^3i + 2tj,\ 0 \leq t \leq 1\).

\[\pi\]