(1) Consider the vector field \( F(x, y, z) = (y - z, x - z, z - y - x) \).

(a) Compute \( \int_C F \cdot d\mathbf{s} \) where \( C \) is the curve \( r(t) = (t, t, t) \), \( 0 \leq t \leq 1 \).

(b) Compute \( \int_C F \cdot d\mathbf{s} \) where \( C \) is the curve \( r(t) = (t, t^2, t^3) \), \( 0 \leq t \leq 1 \).

(c) Do you think the vector field is conservative? If so, find a potential. If not, prove that it is not.
(2) Imagine you are a flightless bug sitting in one corner of a perfectly cubic room. You decide your corner of the room is uncomfortable and you think the corner that is directly opposite you (the one along the longest diagonal of the cubic room) looks more comfy. What is the shortest path from your uncomfortable corner to the more comfy corner? Prove that this is the shortest path.

(3) You only have access to a bucket, a 3-gallon jug, a 5-gallon jug and a hose which dispenses water. How do you get exactly 4 gallons of water into the bucket?

(4) (This riddle stolen from XKCD): A group of people with assorted eye colors live on an island. They are all perfect logicians – if a conclusion can be logically deduced, they will do it instantly. No one knows the color of their eyes. Every night at midnight, a ferry stops at the island. Any islanders who have figured out the color of their own eyes then leave the island, and the rest stay. Everyone can see everyone else at all times and keeps a count of the number of people they see with each eye color (excluding themselves), but they cannot otherwise communicate. Everyone on the island knows all the rules in this paragraph.

On this island there are 100 blue-eyed people, 100 brown-eyed people, and the Guru (she happens to have green eyes). So any given blue-eyed person can see 100 people with brown eyes and 99 people with blue eyes (and one with green), but that does not tell him his own eye color; as far as he knows the totals could be 101 brown and 99 blue. Or 100 brown, 99 blue, and he could have red eyes.

The Guru is allowed to speak once (let’s say at noon), on one day in all their endless years on the island. Standing before the islanders, she says the following:

"I can see someone who has blue eyes."

Who leaves the island, and on what night?

There are no mirrors or reflecting surfaces, nothing dumb. It is not a trick question, and the answer is logical. It doesn’t depend on tricky wording or anyone lying or guessing, and it doesn’t involve people doing something silly like creating a sign language or doing genetics. The Guru is not making eye contact with anyone in particular; she’s simply saying "I count at least one blue-eyed person on this island who isn’t me."

And lastly, the answer is not "no one leaves."

(5) (Credit to Greg Michel for telling me this riddle): Imagine you have a checkerboard (8 squares by 8 squares) and 32 dominoes. The dominoes are such that a single domino will perfectly cover two squares on the checkerboard. There are many ways to cover the checkerboard with the 32 dominoes.

If I remove two opposite corners of the checkerboard (so there are now only 62 squares) is there a way to cover this new checkerboard with 31 dominoes? If so, how? If not, why not?