Abstract:

This thesis is concerned with the efficient evaluation of quasi-periodic Green functions $G_{qp}$, such as those that arise in the modeling and simulation of electromagnetic/acoustic scattering off (infinite) rough surfaces. The difficulties in such evaluations have been long recognized and a variety of techniques have been devised to allow for simulations of scientific and/or technological significance. These techniques, however, are unsuitable for evaluations at high frequencies either due to their computational expense or to their instability or, in some cases, to both. Here we provide a novel numerical algorithm that enables the accurate and efficient evaluation of these Green functions at unprecedented frequencies. More precisely, this new scheme is based on the use of some exact integrals that arise on judicious manipulation of the integral representation of $G_{qp}$. As we show, this can be used to reduce the overall integration problem to a sequence of integrals that can be effectively handled by standard quadrature formulas. A full derivation of the new procedure is presented, and numerical results are included that show that the procedure compares favorably with alternative methods.

In addition to these developments on two-dimensional Green functions, we further present extensions to (a) the evaluation of their partial derivatives; and (b) that of their fully three-dimensional analogues. The former are necessary, for instance, in relation with integral-equation formulations of the scattering problem that involve double-layer potentials; the latter, in turn, are relevant to situations wherein the configuration does not present an invariant direction.

Finally, we also provide a derivation of a spectrally accurate numerical scheme for the solution of quasi-periodic acoustic scattering problems in two space dimensions. A variety of numerical results are presented that confirm the expected convergence properties of the procedure. Although the results contained herein correspond to problems at relatively low frequencies, they suggest a promising performance for shorter wavelengths, especially when coupled to the aforementioned developments on Green functions and to the recent results by Bruno, Reitich, Geuzaine (2004—05) on high-frequency scattering off bounded obstacles; this latter coupling, however, is left for future work.