Adding carbon to conceptual models: an introduction to Hogg's model and others.

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Motivation

- Budyko has only ice albedo feedback.
- “However, amplitude of the glacial cycles cannot be explained by orbital cycles alone”
- We need a feedback mechanism!
- So we introduce a new style of simple model: Hogg’s Model.

Hogg’s Model

- C02 could provide the extra amplitude in the glacial cycles.
- Milankovitch cycles are the trigger for the glacial cycles.

Blackbody Radiation

$$\bar{S} = \sigma \bar{T}^4$$

$$\Rightarrow \bar{T} = 255K \text{ (which is about } 33K \text{ lower than present day Temp).}$$

Stefan-Boltzman Constant $\sigma = 5.67x10^{-8} W/m^2/K^4$.
$\bar{S} \approx 240 W/m^2$

Blackbody Radiation

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Adding Greenhouse Gases may make up the difference

Stefan-Boltzman Constant $\sigma = 5.67x10^{-8} W/m^2/K^4$.
$\bar{S} \approx 240 W/m^2$

Blackbody Radiation

Blackbody Radiation with Greenhouse Gasses:

$$\bar{S} + \bar{G} = \sigma \bar{T}^4$$

Then $\bar{G} = 155W/m^2$ yields the desired $\bar{T} \approx 288K$.

The radiation equation with time dependence becomes:

$$c\frac{dT}{dt} = S + G - \sigma T^4$$

c = specific heat of the ocean = $1.7 \times 10^{10} J/m^2/K$.

Blackbody Radiation

$$c\frac{dT}{dt} = S + G - \sigma T^4$$

$$S(t) = \bar{S} + \Sigma_i S_i sin\left\{\frac{2\pi t}{\Gamma_i}\right\}$$

S_i = amplitude of insolation perturbations = $.25W/m^2$
Γ_i = period of variations in earth's orbit = $10^5 yr$

$$G(t) = \overline{G} + A\ln\left\{\frac{C(t)}{C_0}\right\}$$

$C(t)$ = atmospheric concentration of $C0_2$
C_0 = preindustrial concentration = $280ppm$
A = effect of $C0_2$ on radiation budget = $5.35W/m^2$

Blackbody Radiation

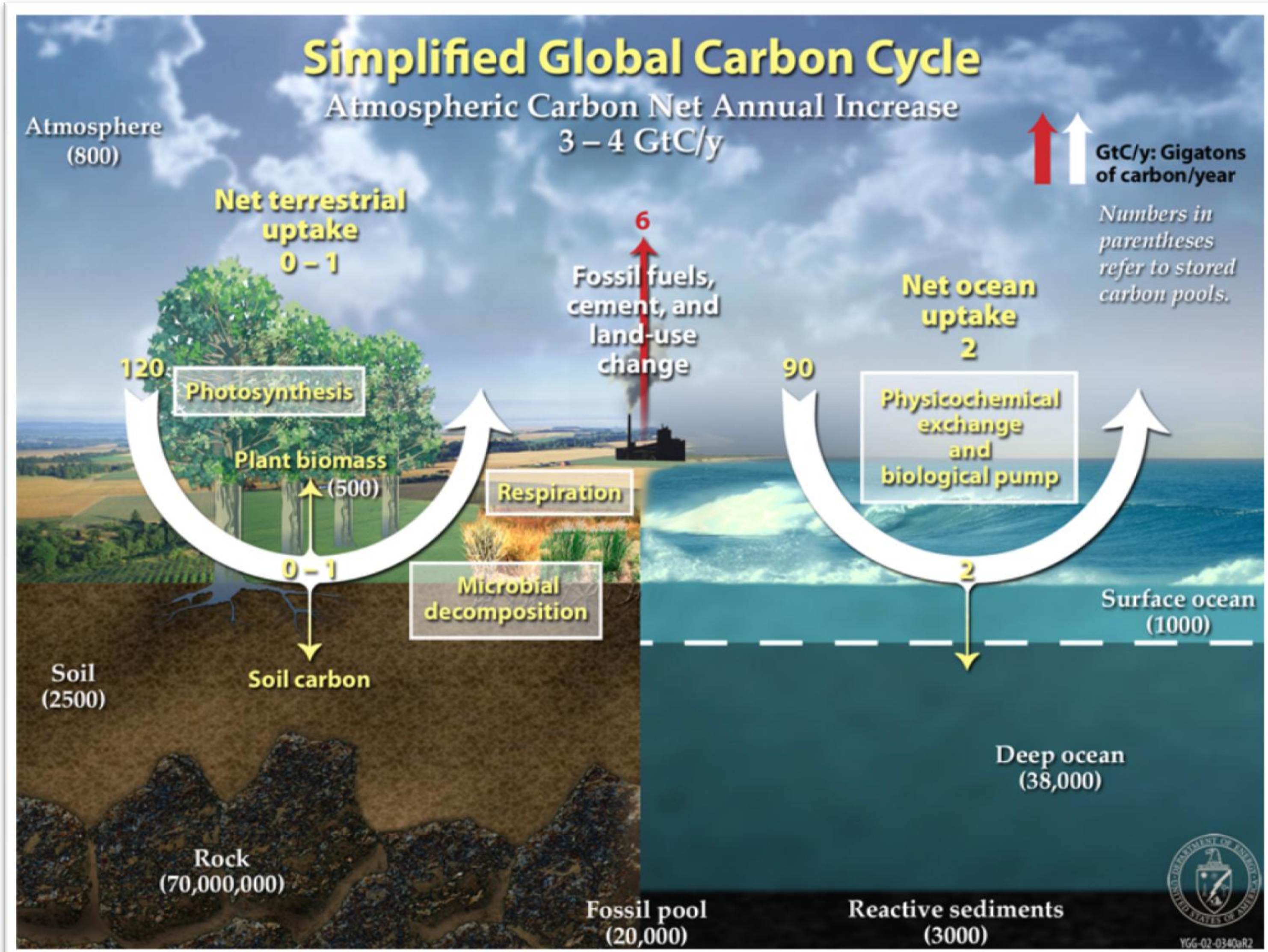
$$c\frac{dT}{dt} = S + G - \sigma T^4$$

$$c\frac{dT}{dt} = \bar{S} + \Sigma_i S_i \sin\left\{\frac{2\pi t}{\Gamma_i}\right\} + \overline{G} + A\ln\left\{\frac{C(t)}{C_0}\right\} - \sigma T^4$$

Atmospheric concentration of CO_2

Now we need to develop a
Atmospheric Carbon Differential Equation.

Atmospheric concentration of CO_2



Atmospheric concentration of CO_2

$$\frac{dC}{dt} = V - (W_0 + W_1 C) + \beta(C_{max} - C)\max\left(\frac{d\bar{T}}{dt} - \epsilon_H, 0\right)$$

Atmospheric concentration of CO_2

$$\frac{dC}{dt} = V - (W_0 + W_1 C) + \beta(C_{max} - C)\max\left(\frac{d\bar{T}}{dt} - \epsilon_H, 0\right)$$

constant source of CO_2 due to volcanoes
estimated at 0.018-0.03 ppm/yr (*Gerlach, 1991*)

Atmospheric concentration of CO_2

$$\frac{dC}{dt} = V - (W_0 + W_1 C) + \beta(C_{max} - C)\max\left(\frac{d\bar{T}}{dt} - \epsilon_H, 0\right)$$

carbon contributed to ocean through weathering of silicate rocks

W_0 = 0.013 ppm/yr and W_1 = 12,000 yr (*Toggweiler, 2007*)

Atmospheric concentration of CO_2

$$\frac{dC}{dt} = V - (W_0 + W_1 C) + \beta(C_{max} - C)\max\left(\frac{d\bar{T}}{dt} - \epsilon_H, 0\right)$$

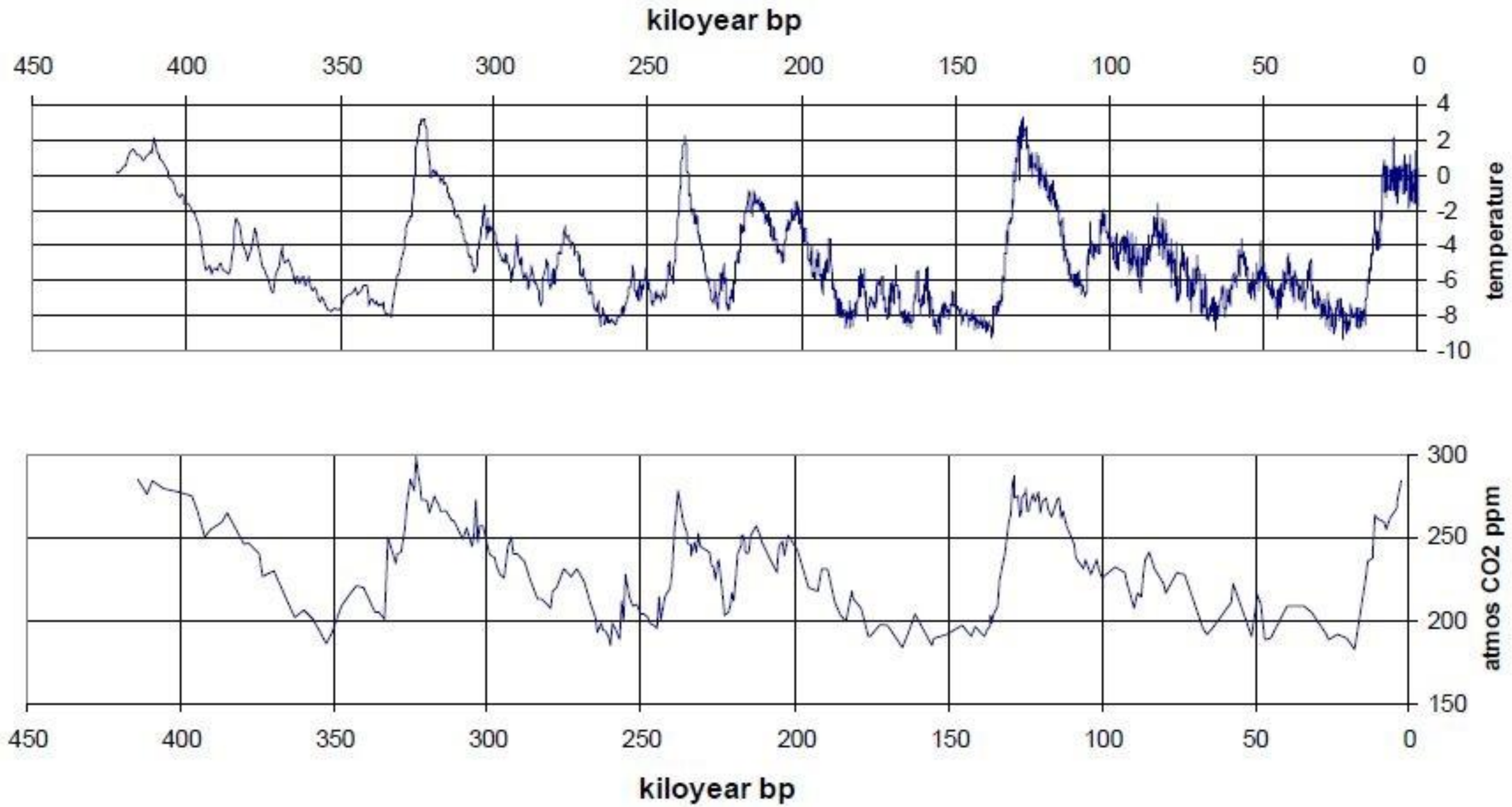
- release of CO_2 with significant warming; C_{max} limited by amount of oceanic CO_2 readily available

C_{max} = 400ppm

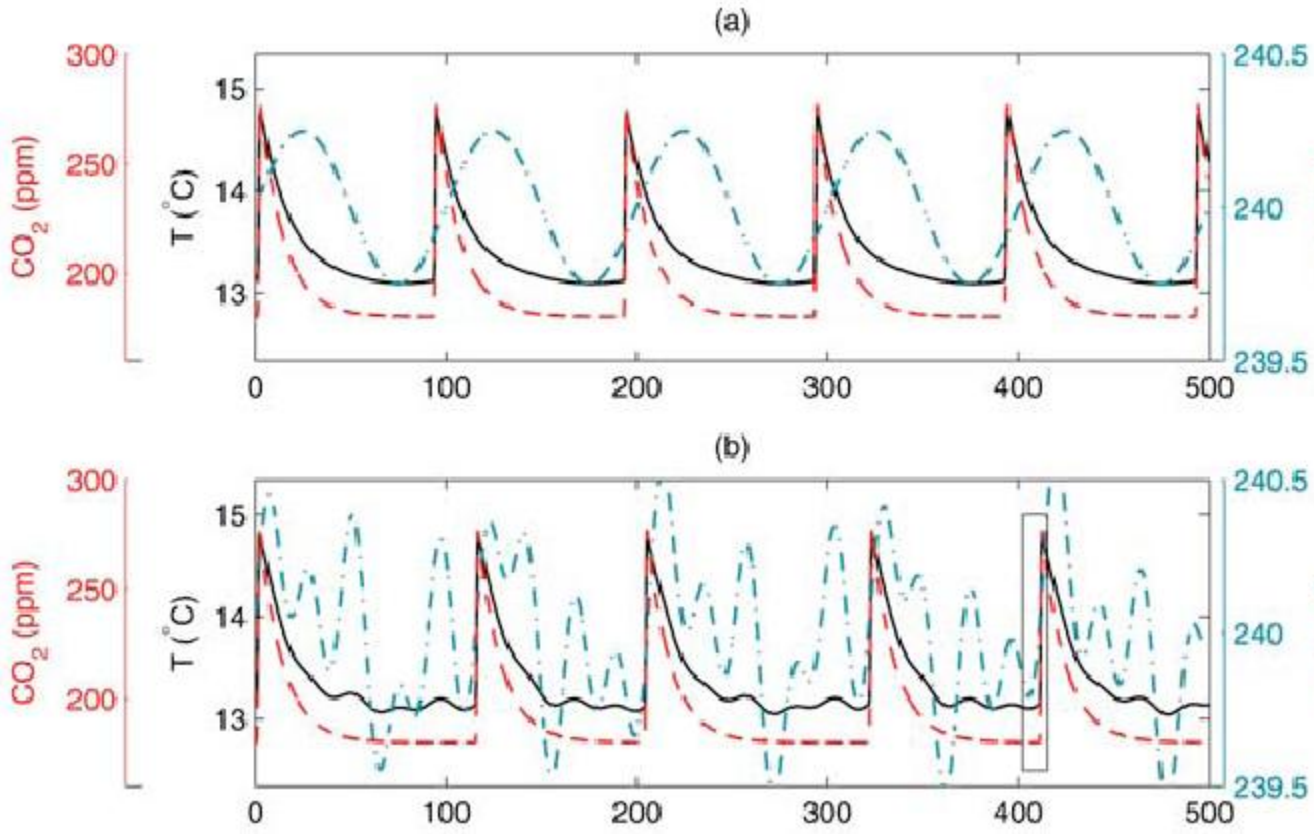
Beta = 0.38^{o}C^-1

Vostok Core Sample Data

Petit, et al, *Nature* **399** (June 3 1999), pp.429-436

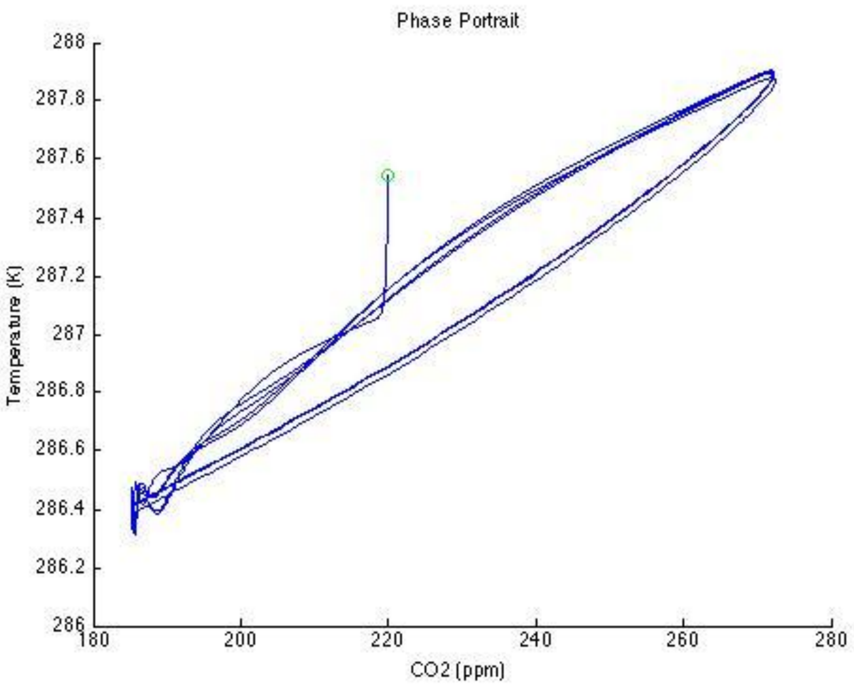


Results

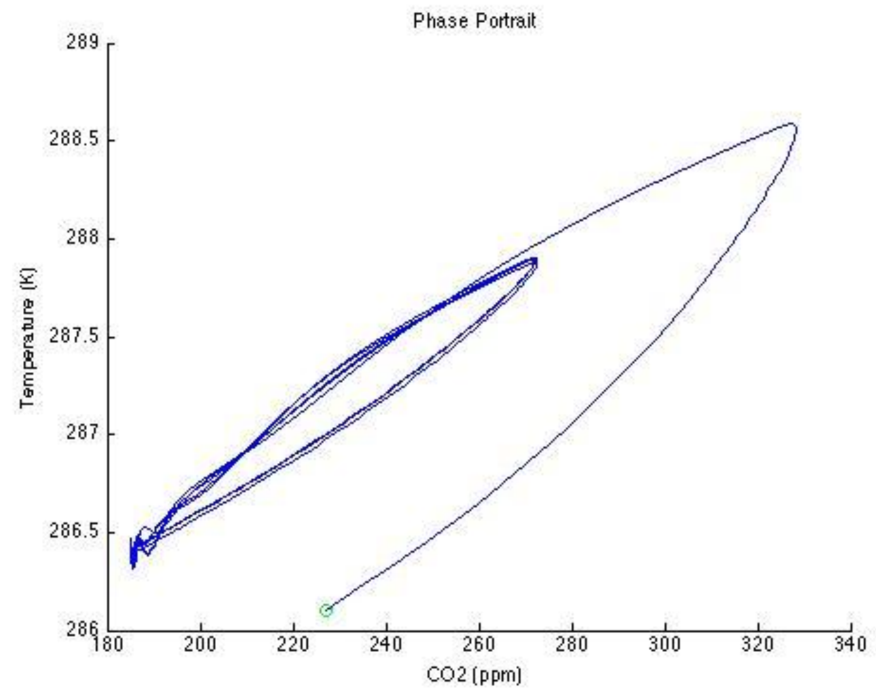


Obliquity – 41k
Precession- 23k
Eccentricity- 100k

Changing Temperature

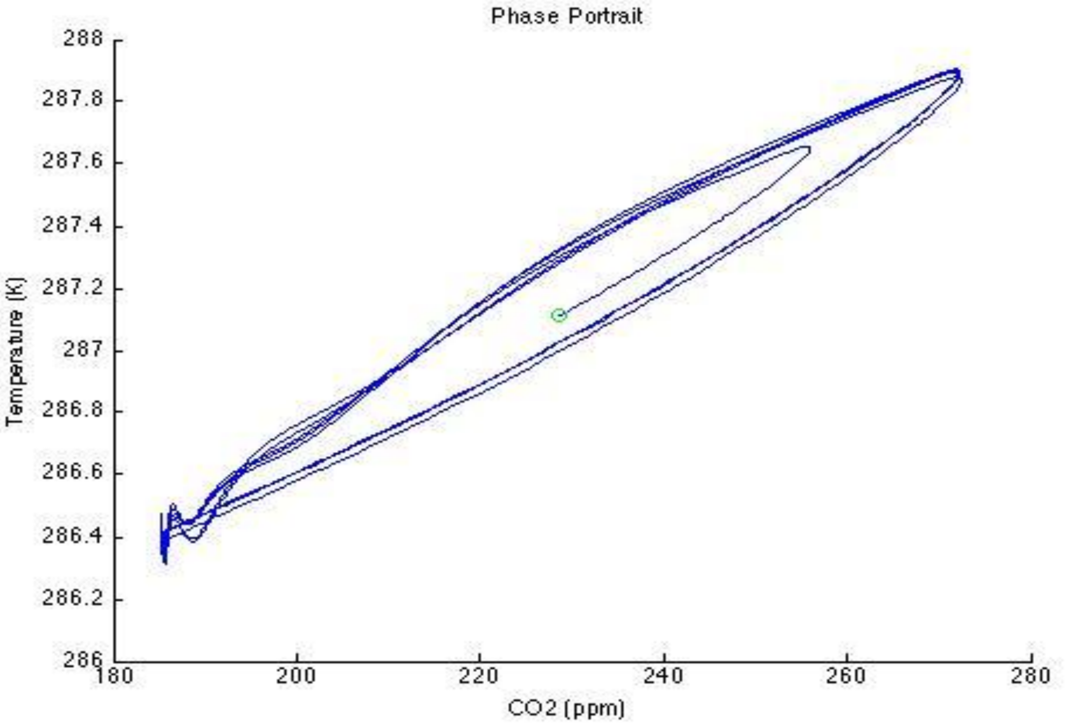


Initial Conditions:
CO_2: 220 ppm
Temp: 287.6 K



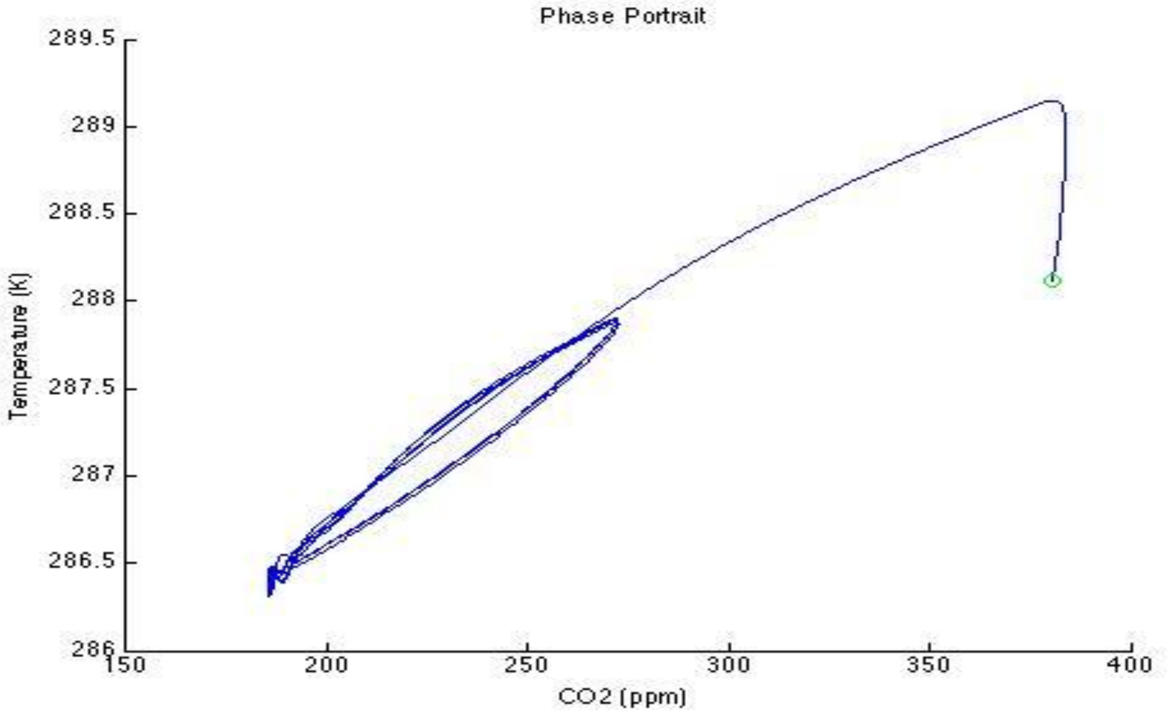
Initial Conditions:
CO_2: 220 ppm
Temp: 286 K

Starting inside the limit cycle



Initial Conditions:
CO_2: 228 ppm
Temp: 287.1 K

Current Conditions



Initial Conditions:
CO_2: 380 ppm
Temp: 288 K

Hogg Conclusions

Temperature ranges exceeds that due to insolation alone.

Temporal response is asymmetric due to large outgassing of CO_2 produced by global warming.

“This modeling approach is proposed as an important tool in distinguishing between proposed mechanisms for control of glacial cycles.”

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"This modeling approach is proposed as an important tool in distinguishing between proposed mechanisms for control of glacial cycles."

Other Carbon Models

Budyko-Sellers-Widiasih

variables: temperature and ice line

Hogg

variables: temperature and atmospheric CO_2

Maasch and Saltzmann

variables: ice volume, atmospheric CO_2, and deep water salinity/temp

Boulder "Awesome" Model

variables: temperature, ice line, and atmospheric CO_2.

Maasch and Saltzmann

Maasch and Saltzmann

variables: ice volume, atmospheric CO_2, and deep water salinity/temp

$$\dot{I}' = -a_0 I' - a_1 \mu' - a_2 M(t)$$

$$\dot{\mu}' = b_1 \mu' - (b_2 - b_3 N')N' - b_4 N'^2 \mu'$$

$$\dot{N}' = -c_0 I' - c_2 N'$$

I = Global Ice Mass

N = North Atlantic Deep Water NADW

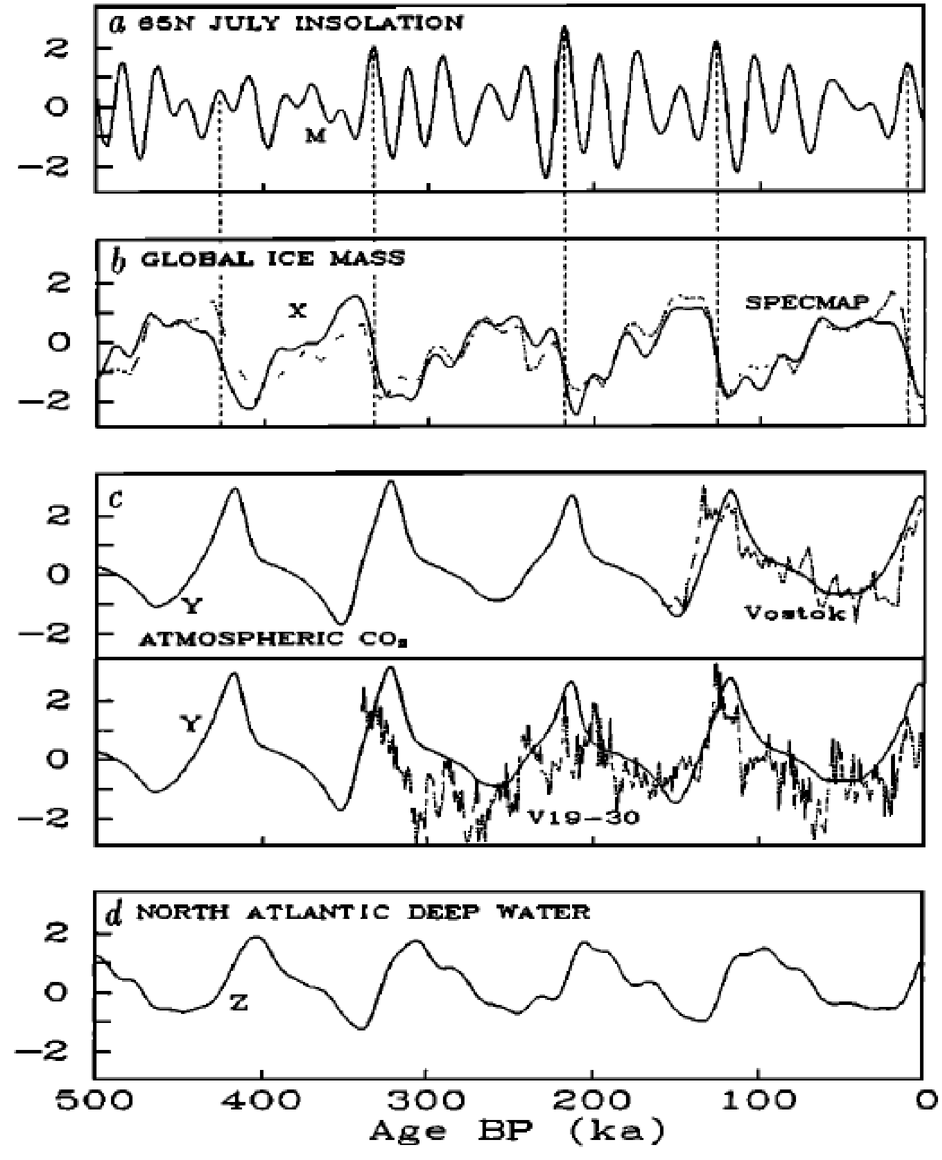
μ = Atmospheric CO_2

Primes denote departures from an equilbrium state controlled by possible ultraslow variation of the solar constant and the tectonic state of the Earth.

$a_{0,1,2}$ $b_{1,2,3,4}$ and $c_{0,2} > 0$

M(t) = Milankovitch Forcing (65° N normalized to 0 mean and unit variance)

Maasch and Saltzmann



Budyko-Sellers-Widiasih Model

$$\frac{\partial T}{\partial t} = \frac{k}{R}(Q \quad s(y) * (1 - \alpha) - (A + BT) + H(\overline{T} - T(y))$$

$$\frac{d\eta}{dt} = k\epsilon(T(\eta) - T_c)$$

Boulder "Awesome" Model

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$$\frac{dC}{dt} = \underbrace{V}_{\text{Volcanoes}} - \underbrace{(W_0 + W_1 C)}_{\text{Weathering}} + \underbrace{\beta(C_{max} - C)max(\frac{d\overline{T}}{dt} - \epsilon_H, 0)}_{\text{Ocean Outgassing}} + \underbrace{k_1\eta + k_2 + k_3\overline{T}}_{\text{Biology + Ocean}}$$

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Volcanoes | Weathering | Ocean Outgassing | Biology + Ocean

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Volcanoes Weathering Ocean Outgassing Biology + Ocean

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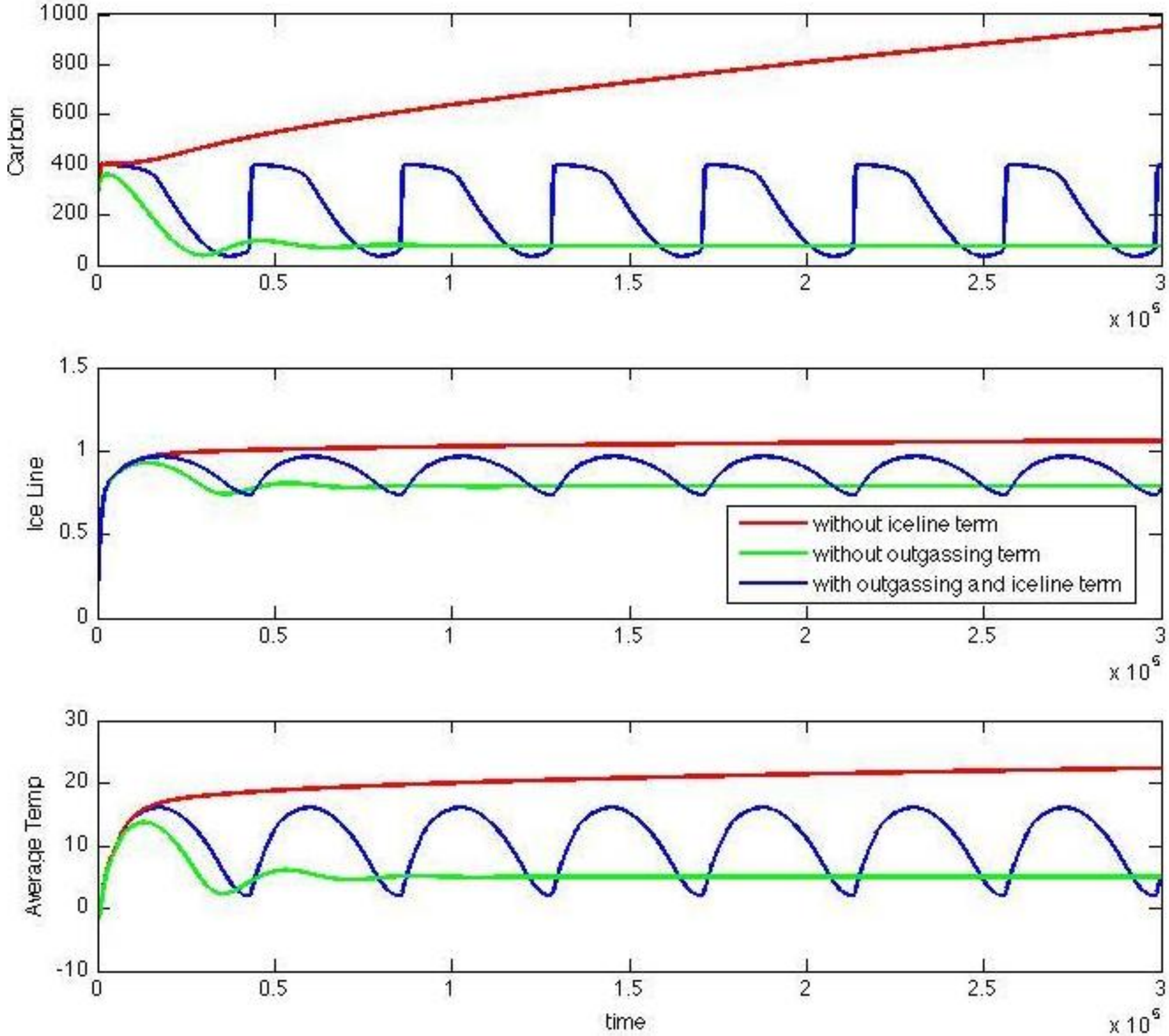
Volcanoes Weathering Ocean Outgassing Biology + Ocean



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Thanks!

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