Underwater Mathematics: Illuminating Deep-Reaching Ocean Eddies in Climate Models

Samantha Oestreicher
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“Some say the world will end in fire…”

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Weather:

Do I need an umbrella *today*?

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Weather:

Do I need an umbrella *today*?

Climate:

Do I need to *own* an umbrella?

Why are Mathematicians involved?

We want to know what’s going to happen to the planet.

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Historic Climate Model: Energy Balance

~~Less~~ Simple Albedo Model

(Budyko, Sellers – Tung version)

energy imbalance = insolation – reradiation + transport

$$R\frac{\partial T}{\partial t} = Qs(y)\big(1-\alpha(y)\big) - (A+BT) + C\left(\bar{T}-T\right)$$

y = sine(latitude)

$T(y,t)$ = annual mean surface temperature at latitude arcsin(y)

$Qs(y)$ = annual mean insolation at latitude arcsin (y)

$\alpha(y)$ = surface albedo at latitude arcsin (y)

$\bar{T}$ = global mean temperature

$C\left(\bar{T}-T\right)$: linear relaxation to mean

$s(y)$ = distribution of insolation across latitudes

$$\int_0^1 s(y)\,dy = 1$$

Choice of y instead of latitude: $\bar{T} = \int_0^1 T(y)\,dy$

“Modern” Climate Model

Global Climate Models (GCM) such as CCSM, the Community Climate System Model

Me in front of Bluefire,
NCAR’s newest super computer.

"Modern" Climate Model

Global Climate Models (GCM) such as CCSM, the Community Climate System Model

1.5 million lines of code

Simulates 2.5 years per day on the fastest computers in the world at 10km length scales

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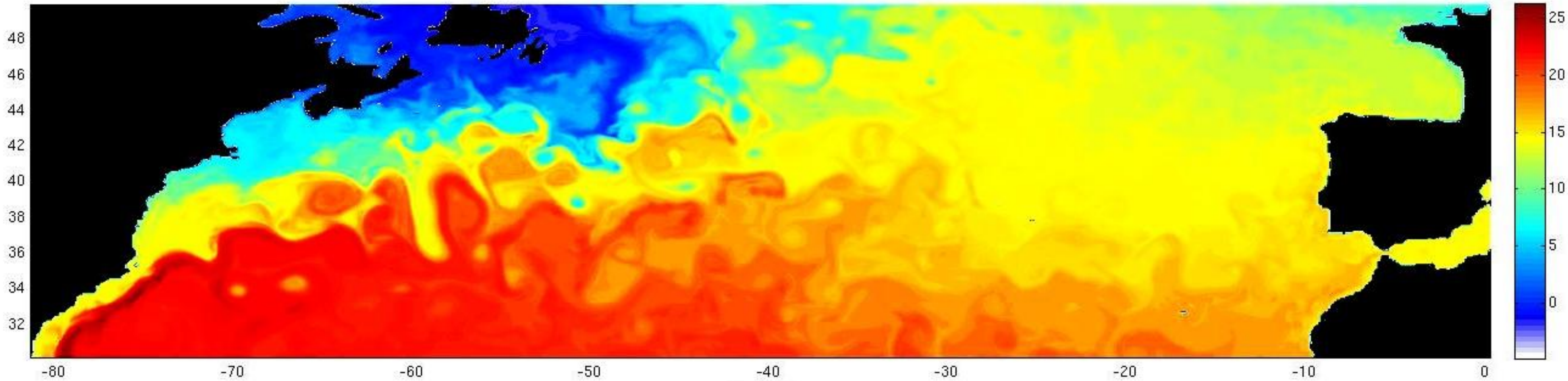
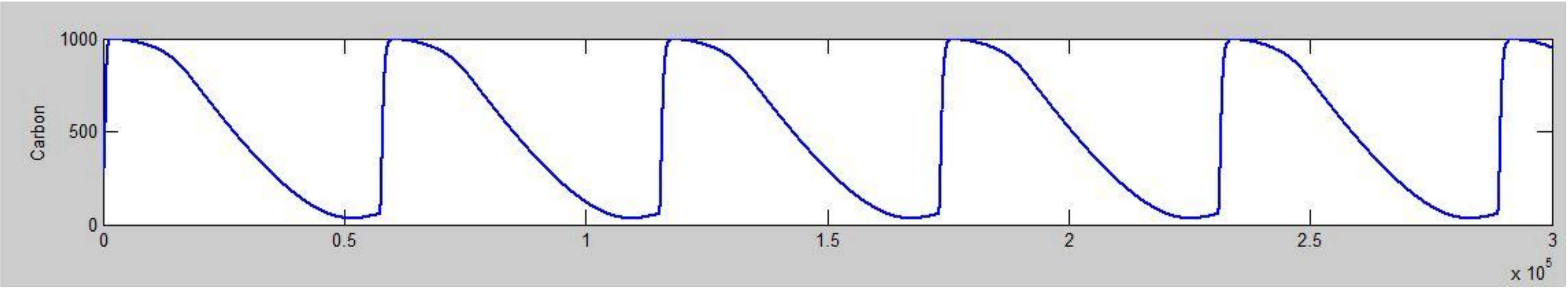
NCAR's newest super computer.

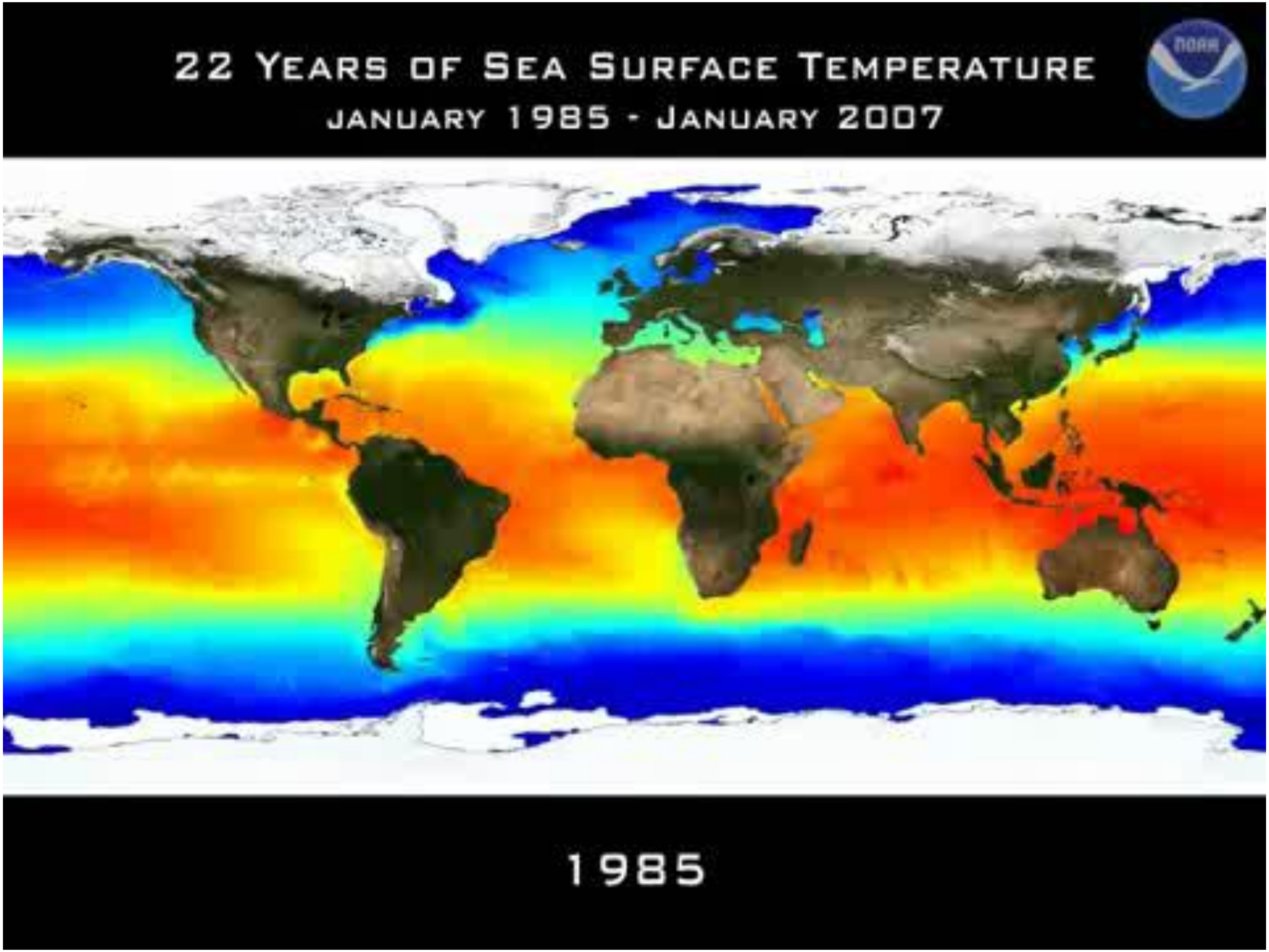
GCM vs. Simple



Model	Grid Size	Simulated Time	Run Time	Math?	Images
Simple	Whole Earth	50,000 yrs	30 seconds	Yes!	Eww.
GCM	10 km	100 yrs	2 months	Maybe?	Gorgeous!



How accurate are GCMs?



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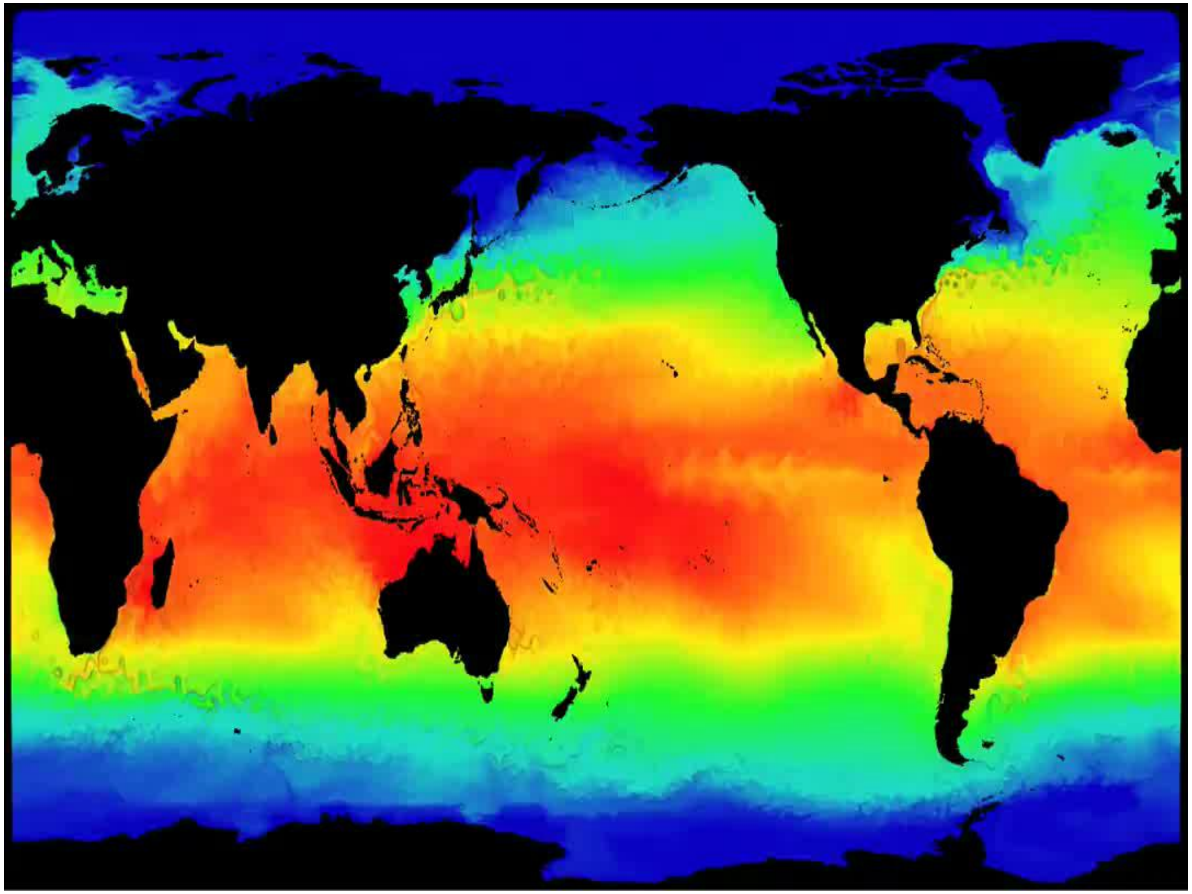
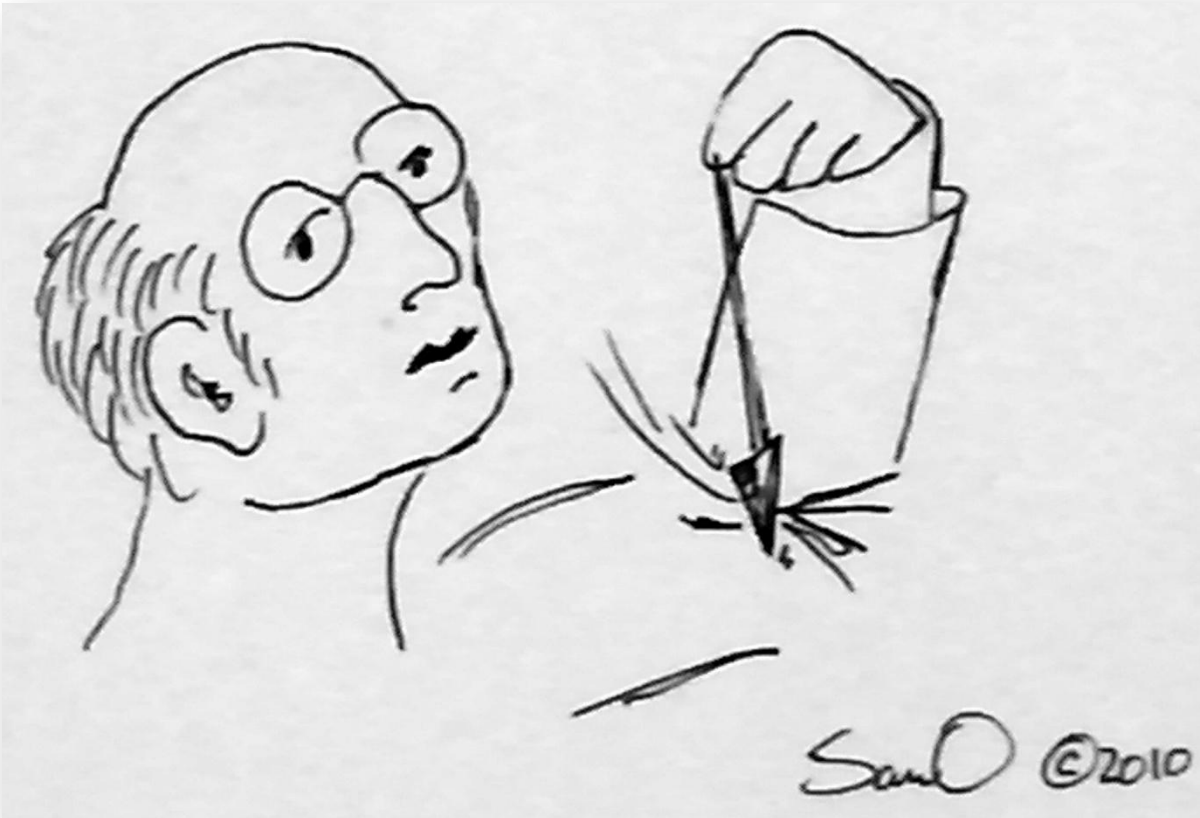


Parallel Ocean Model Output

CCSM

Learning from our Models

Now that our GCMs are so big we can't analyze their structure, we must analyze their output.



Learning from our Models

Now that our GCMs are so big we can't analyze their structure, we must analyze their output.

What can we learn from the model output that we can then confirm with real life observational data?

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<u>Take Two!</u>

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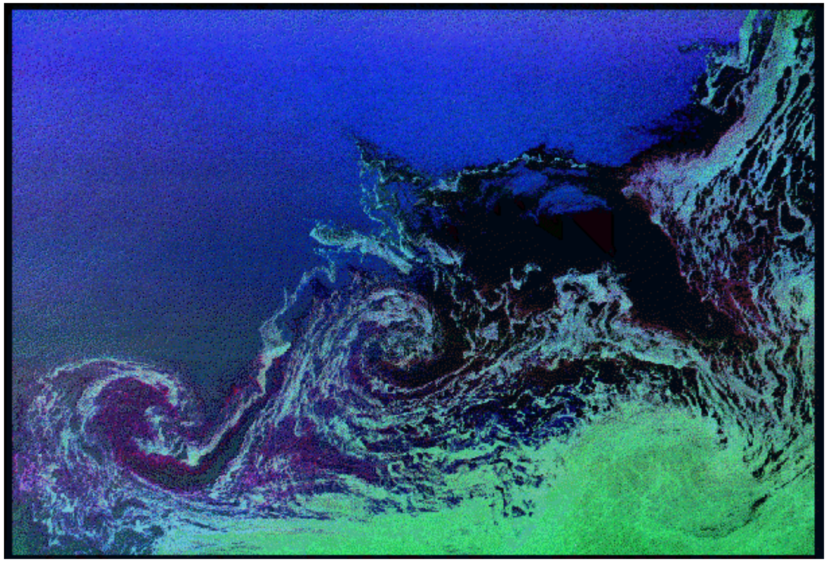
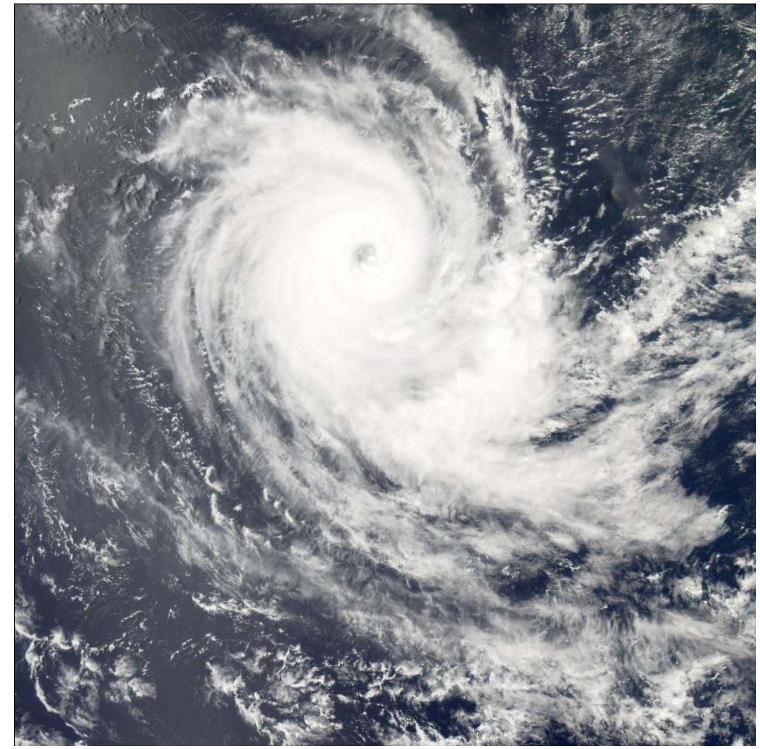
What Is An Eddy?





Ocean Eddy

Atmospheric

Cyclone

http://www.nasa.gov/images/content/143846main_carina_AMO_2006058_lrg.jpg
http://www.clivar.org/organization/wgomd/meso/meso.php

Why Study Eddies?

Eddies exist on many length scales.

Eddies are integral ocean heat and salt mixing.

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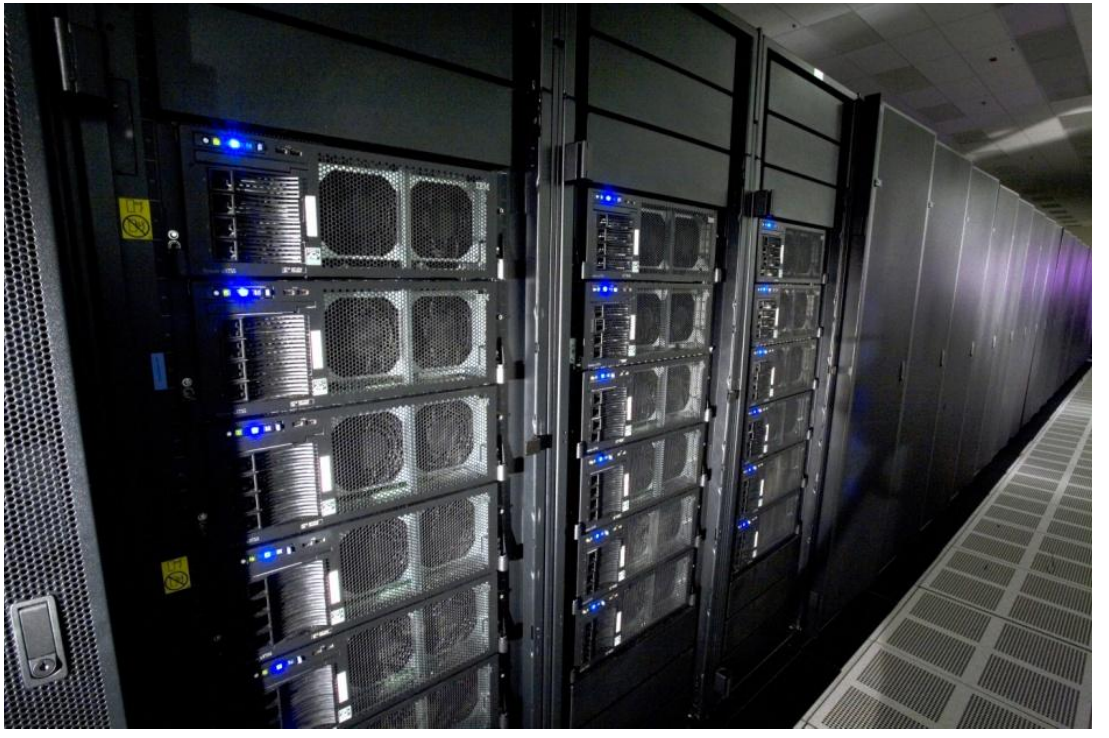
Understanding eddies is vital to predicting climate change.

Models provide a reasonable way observe all depths.

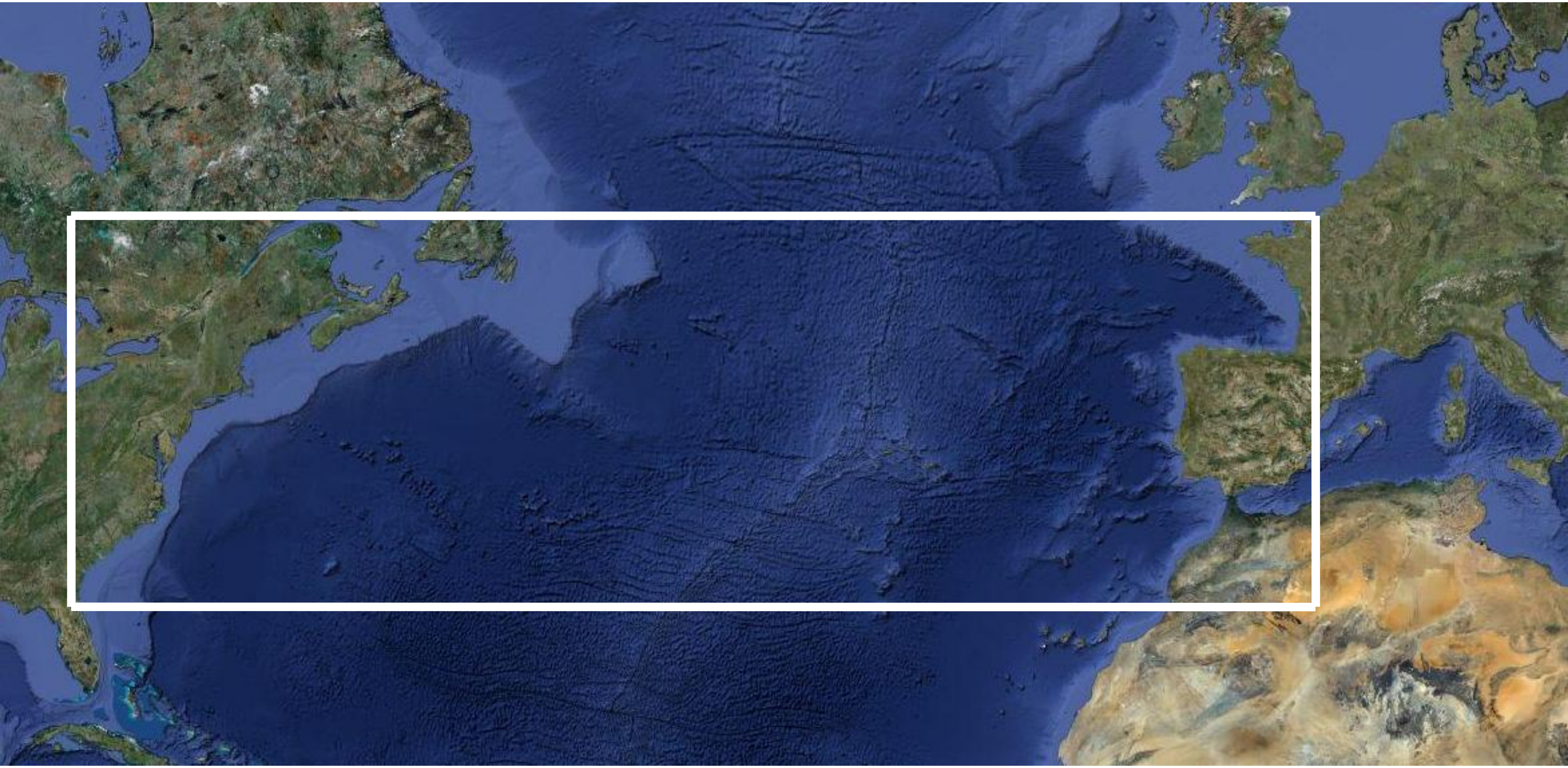
Which Model?



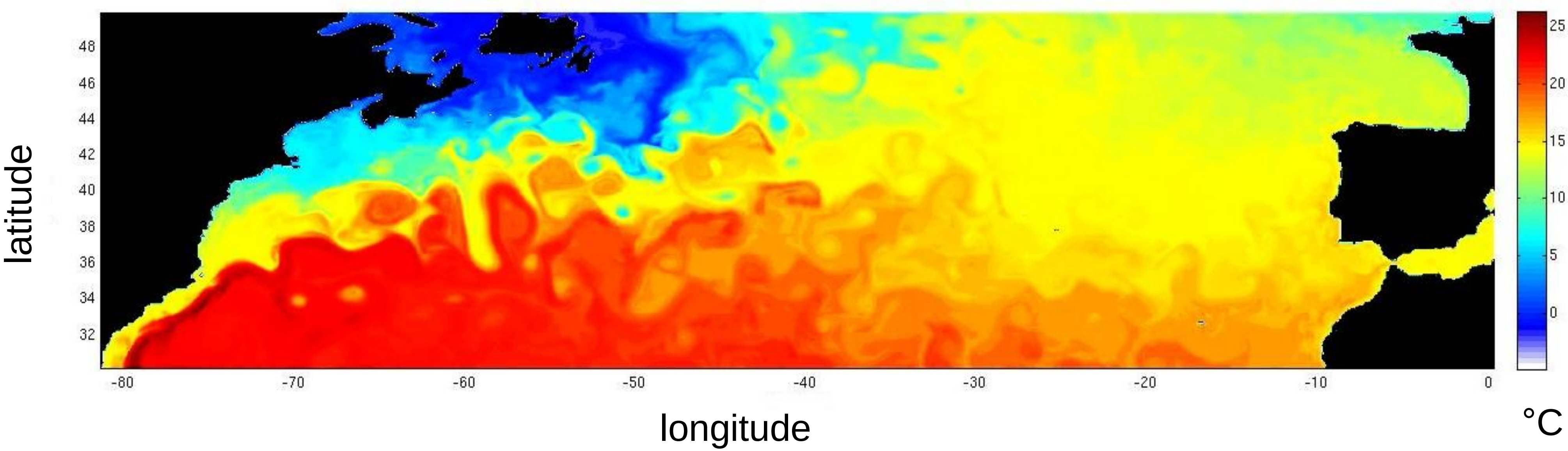
Model Output from Parallel Ocean Program (POP) developed at Los Alamos National Labs (LANL).

POP is a part of Community Climate System Model (CCSM4) used in IPCC Report.

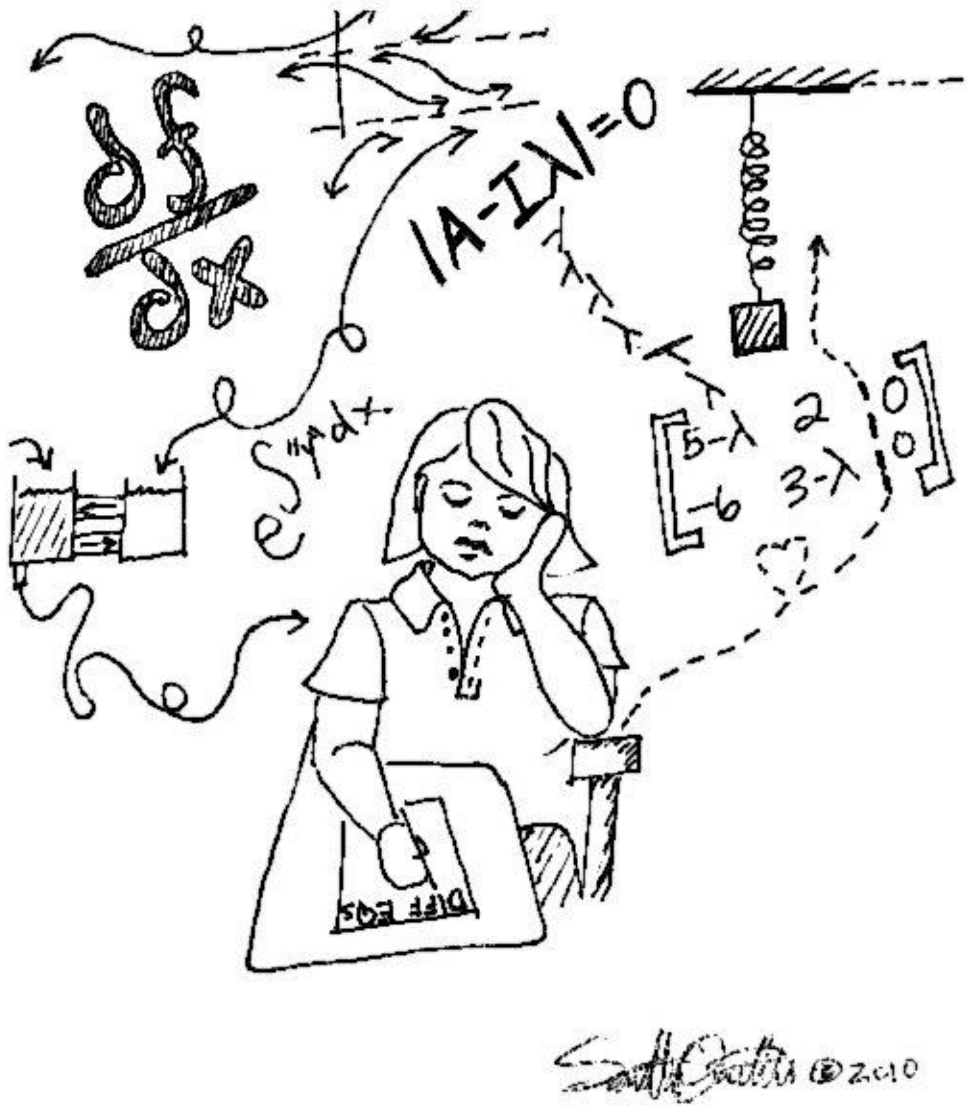
Starting in Northern Altantic



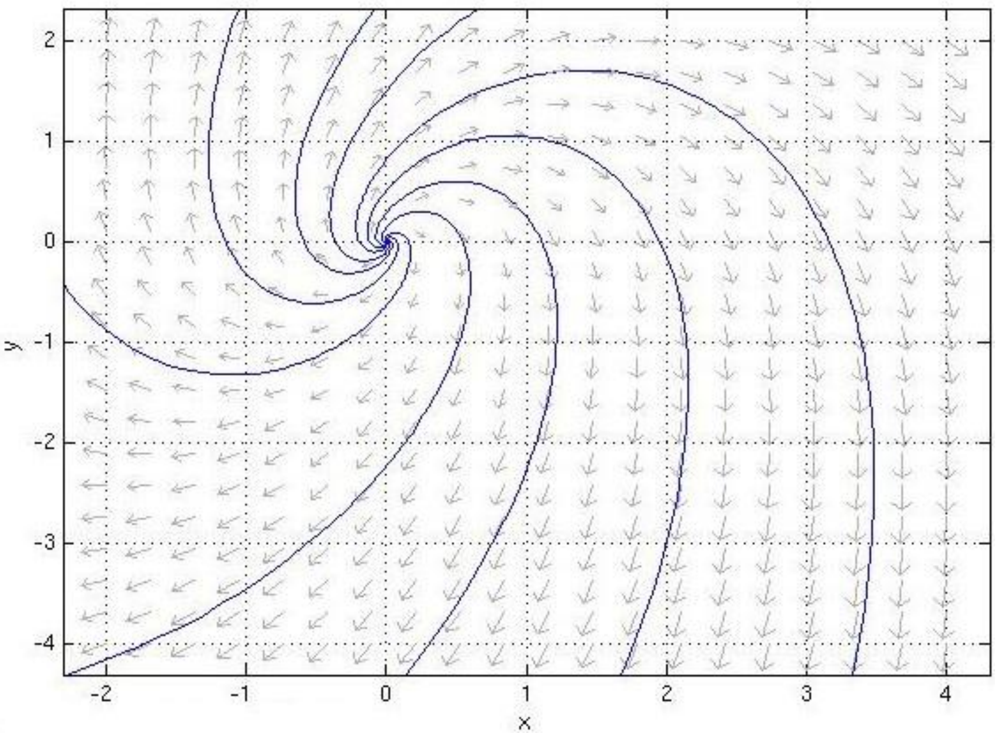
Modeling the Northern Atlantic



Differential Whatnow?

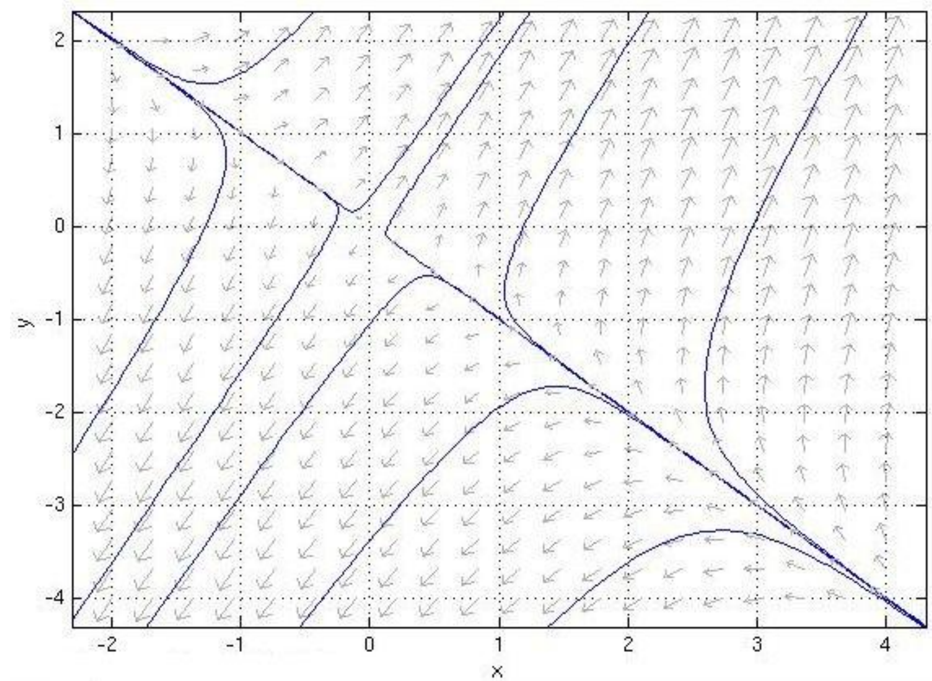


Eigenvalues Classify Flow



x' = 2x + 3y
y' = 6x +5y

Eigenvalues of 3.5 + 3.9686i and 3.5 – 3.9686i



x' = 2x + 3y
y' = -6x +5y

Eigenvalues of 8 and -1

Okubo-Weiss Parameter

V (u,v) = velocity

u(x,y) and v(x,y) = directional velocity

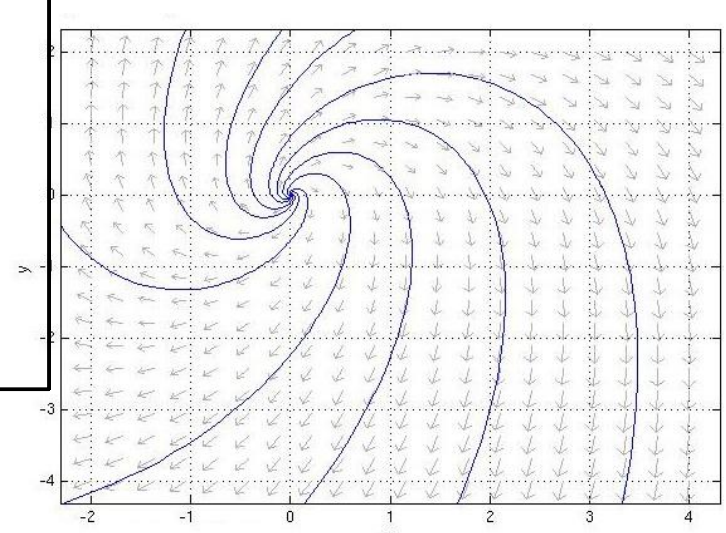
The velocity flow is spiraling

iff

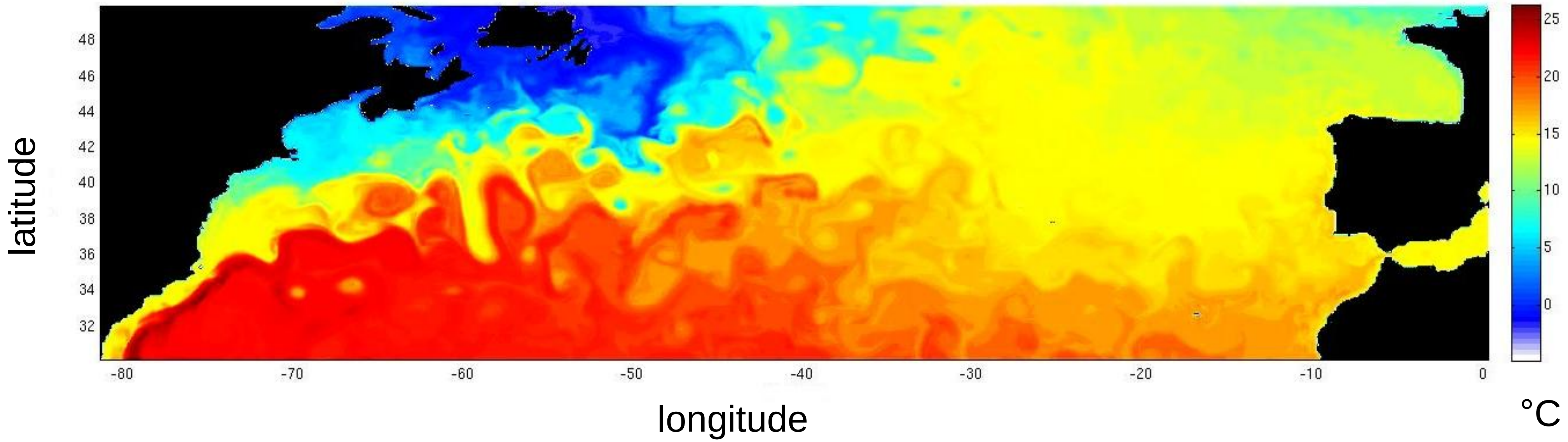
the discriminant of the characteristic polynomial of $\nabla \boldsymbol{V} < 0$

iff

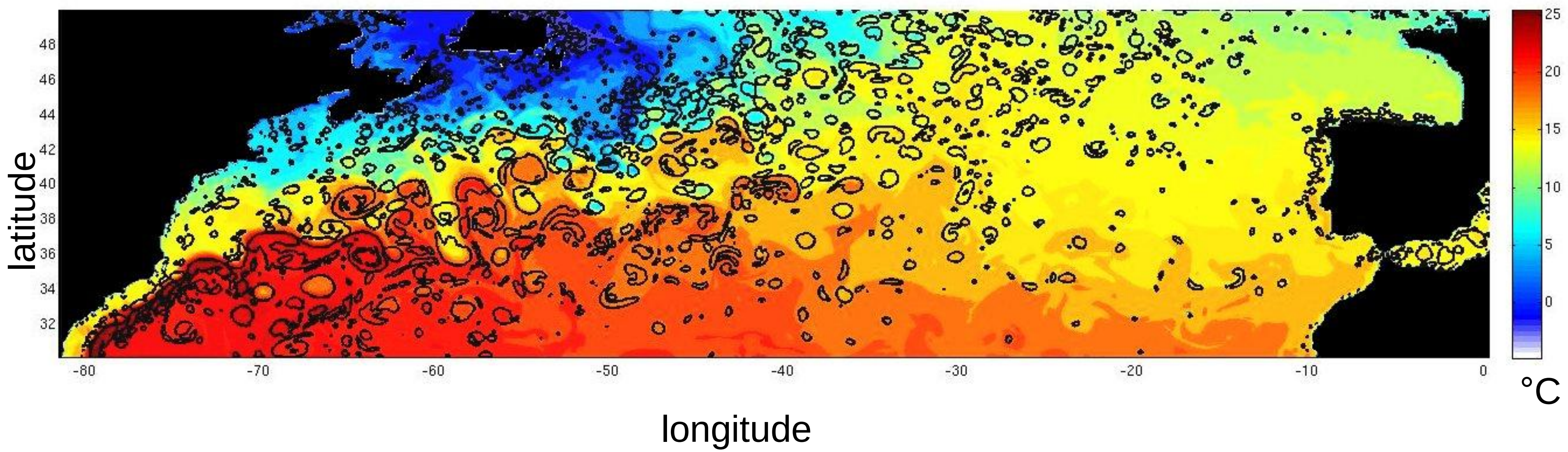
$$\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dy}\right)^2 - 2\frac{du}{dx}\frac{du}{dy} + 4\left(\frac{dv}{dx}\frac{du}{dy}\right) < 0$$



Results



latitude
48
46
44
42
40
38
36
34
32
-80
-70
-60
-50
-40
-30
-20
-10
0
longitude
25
20
15
10
5
0
°C



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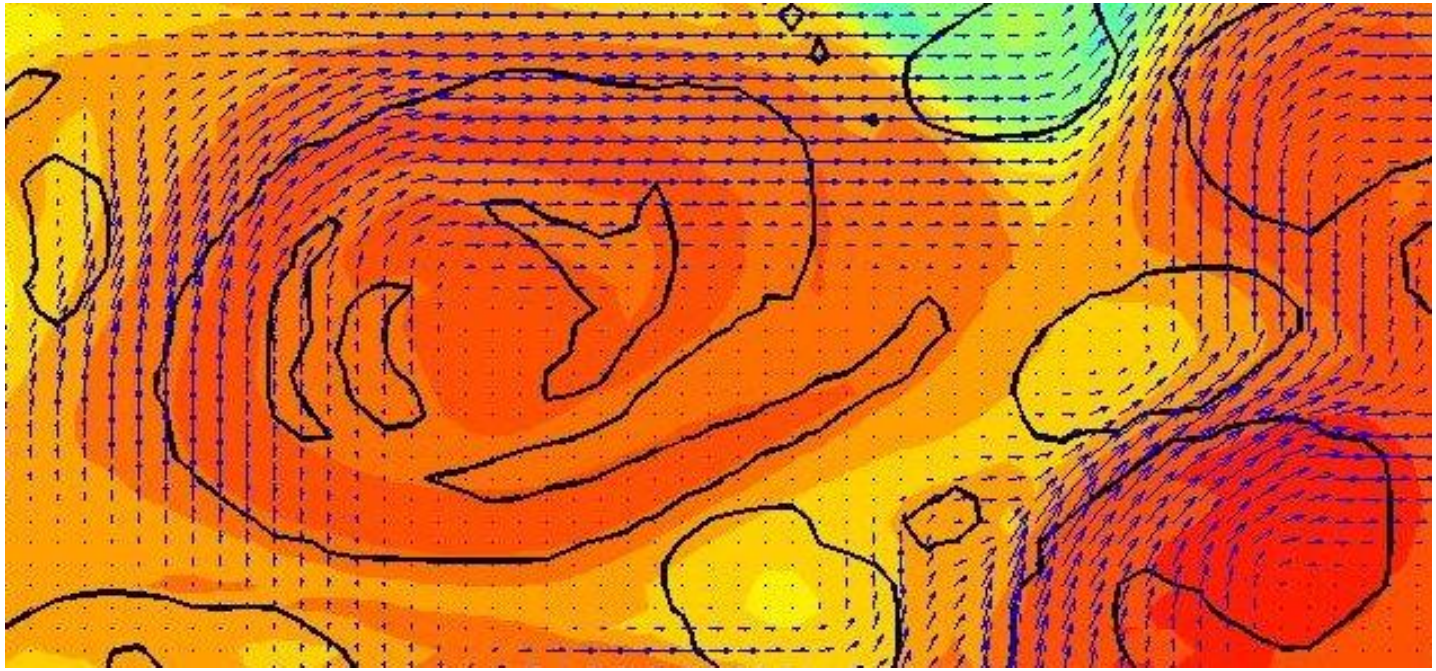
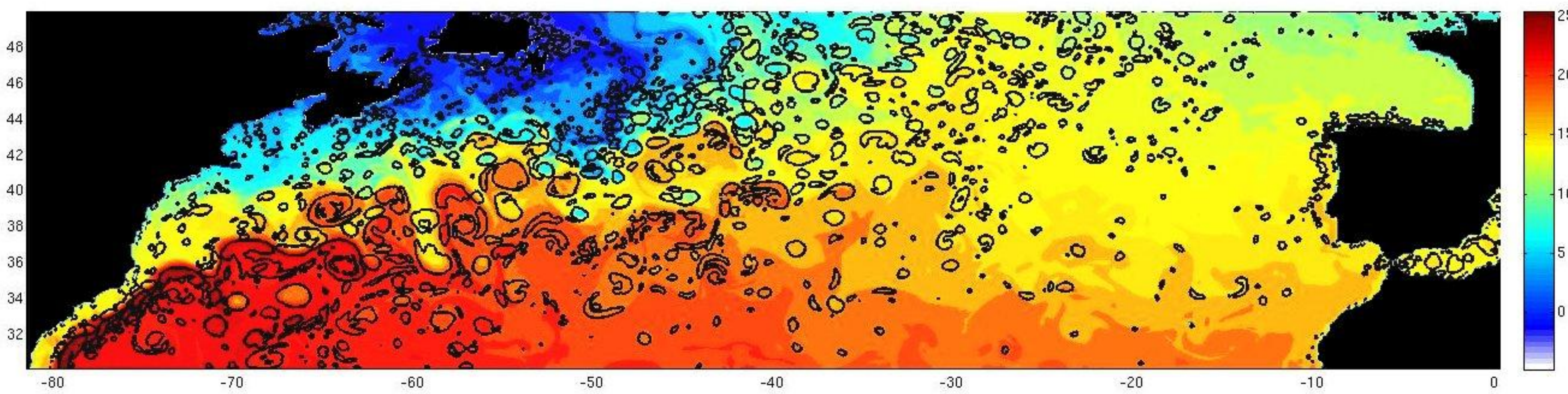




latitude

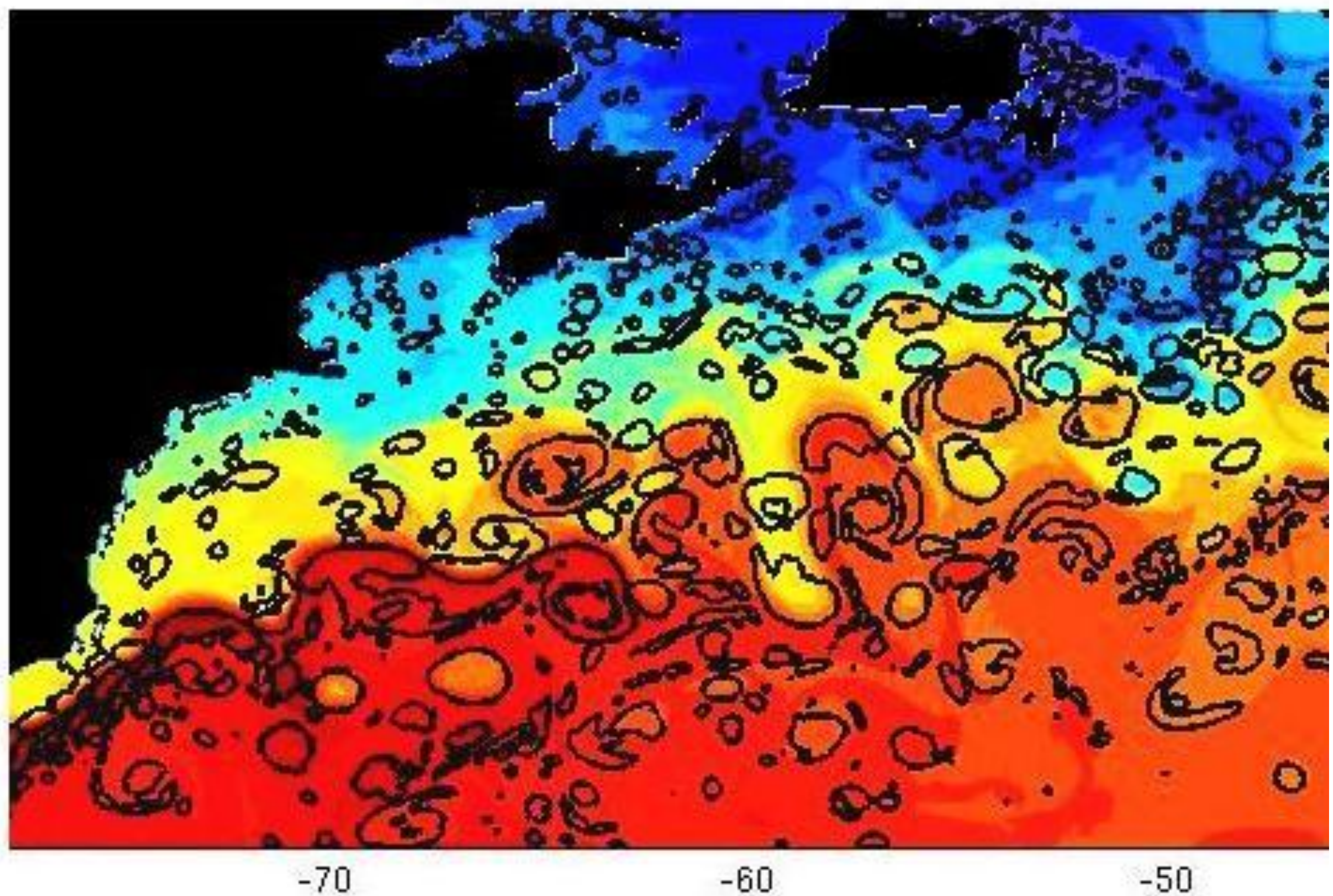
longitude

°C

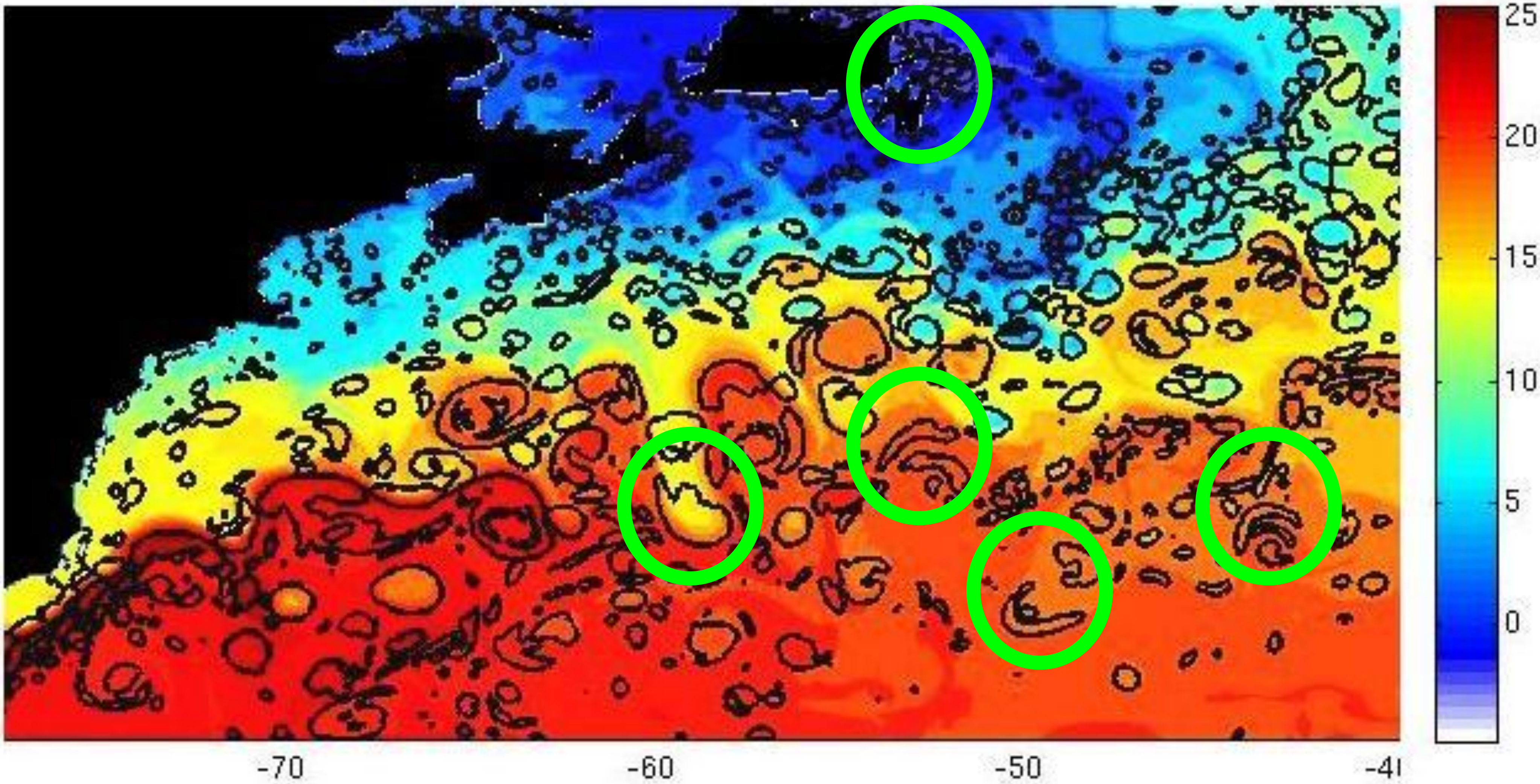
But…

Okubo-Weiss is famous for generating false positives.

Especially with meanders:

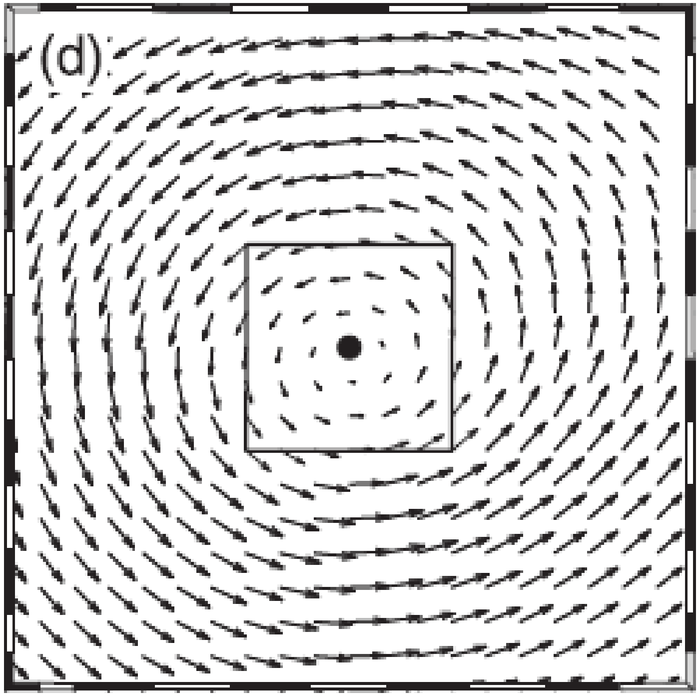


False Positives



Meanders, Strong Flows, and Computational Error

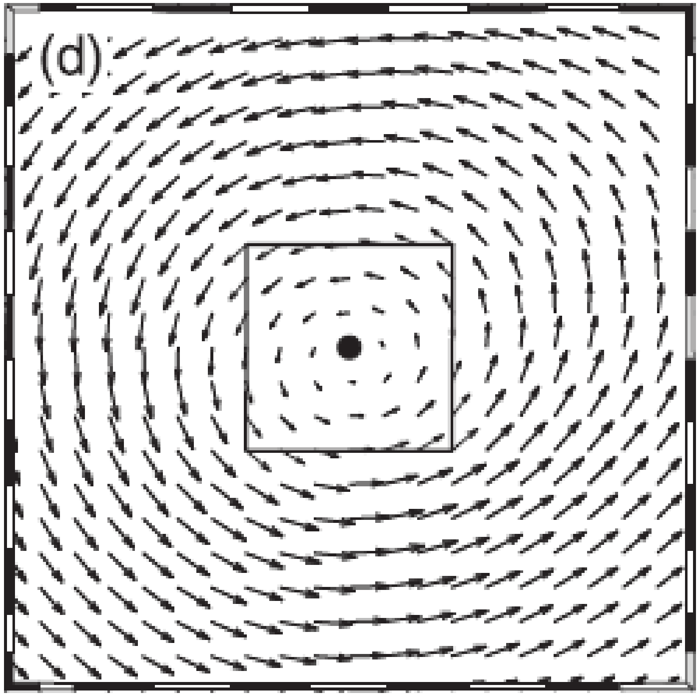
Four Corner Method

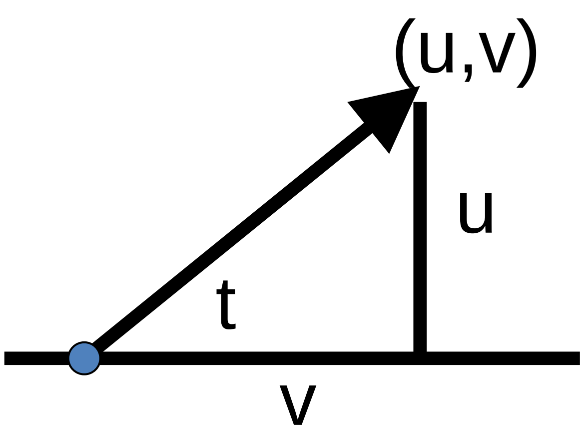


Q: Given velocity vectors u and v, determine the direction of the flow.

Nencioli, Dong, Dickey, Washburn, McWilliams "A Vector Geometry-Based Eddy Detection Algorithm and Its Applications to a High-Resolution Numerical Model Product and High-Frequency Radar Surface Velocities in the Southern California Bight" *Journal of Atmospheric and Oceanic Technology*. 2009. Pg: 564-579
Strelitz, Richard. "Geometrically identifying Eddies" Presentation at CNLS, LANL, Summer 2010.

Four Corner Method





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A: u + v = (u,v).

Q: Given the direction of flow, determine the angle of incidence.

A:

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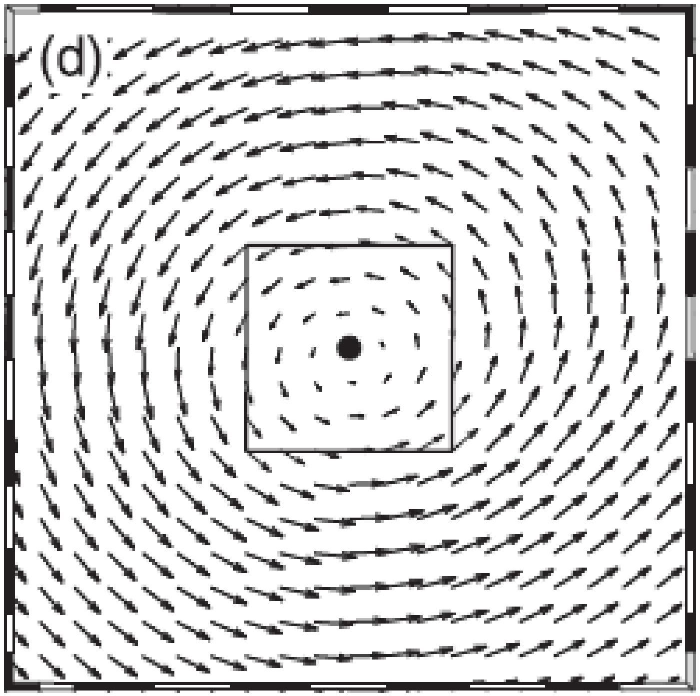

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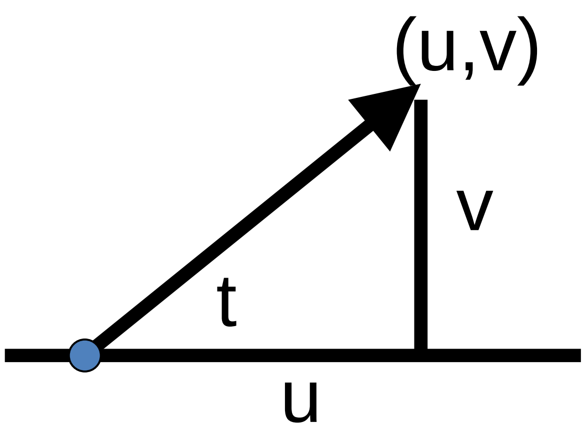
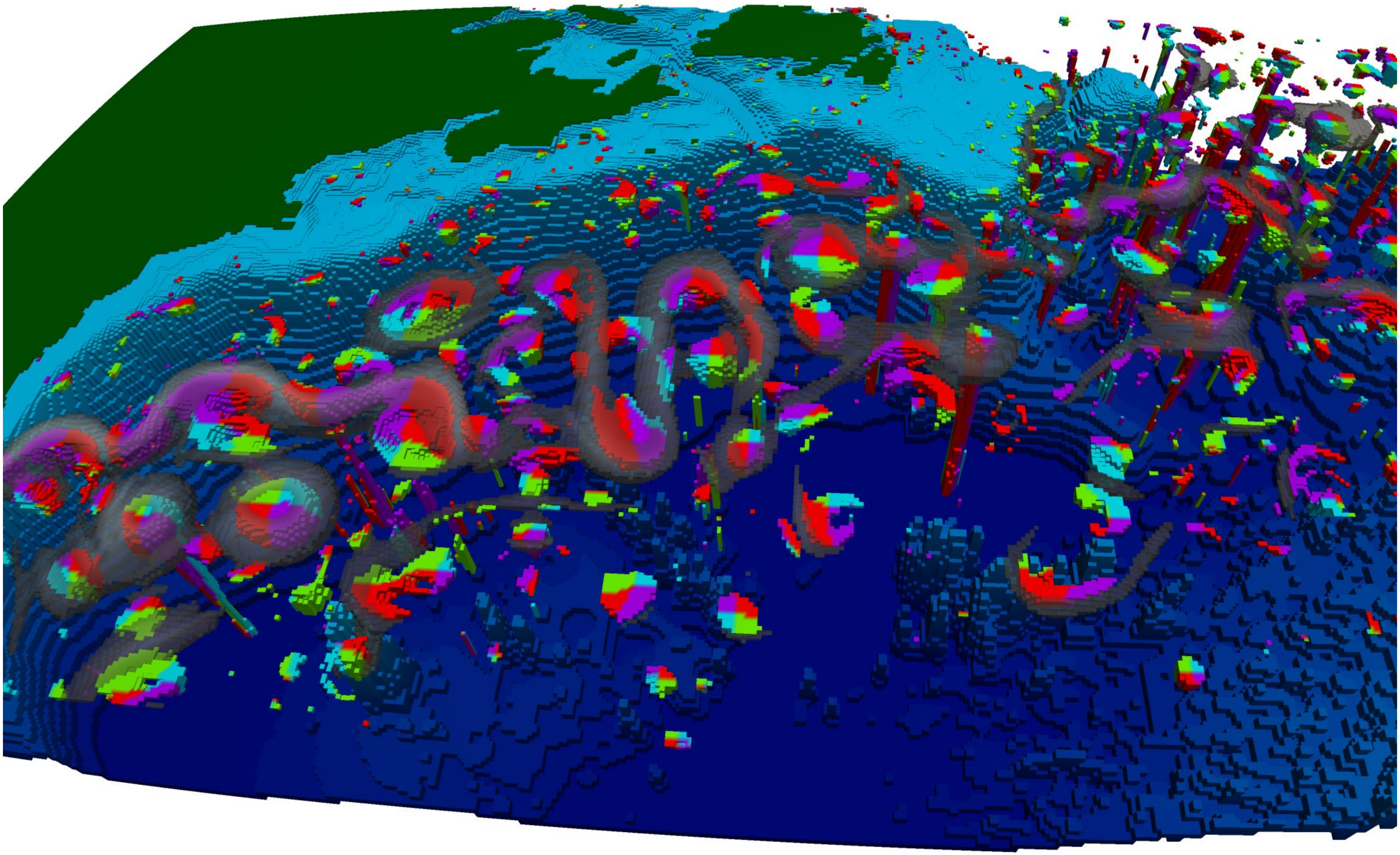


$u/v < 0$ $v > 0$	$u/v > 0$ $v > 0$
$u/v > 0$ $v < 0$	$u/v < 0$ $v < 0$

Four Corner Method



Sean Williams “Feature Extraction and Visualization for Ocean Simulation” LANL Summer 2010

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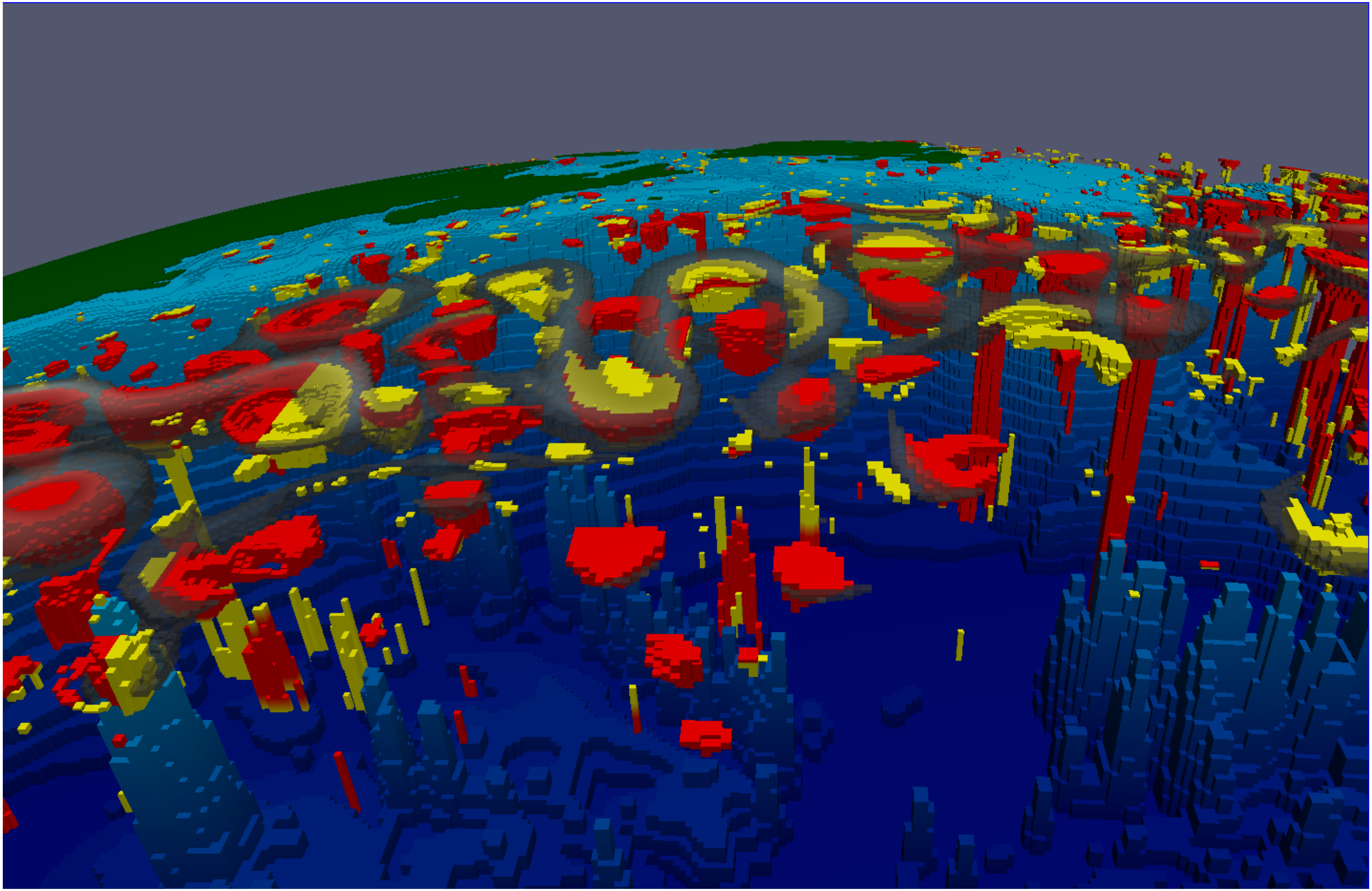

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Conclusions

Three step method for identifying eddies.

Removes most of the false positives from Okubo-Weiss.

Creates a more accurate global census which includes depth and volume analysis.

Future Directions

Eddies exist under the ice sheets.

Eddies are integral to the longevity of the ice caps.

Limited knowledge due to limited observations.

Model results provide a unique way to “see” under the ice.

http://www.sethwhite.org/high%20arctic%20scenery.htm

Arctic Expectations

Arctic Ocean has less vertical stratification in density

Deep Slender Eddies

Vorticity is conserved

aside: Ballerinas also conserve vorticity

http://www.photographyblogger.net/wpcontent/uploads/2009/05/ballerina6.jpg
http://home.earthlink.net/~dmac1137/sitebuildercontent/sitebuilderpictures/pirouette.jpg

Arctic Expectations

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Deep Slender Eddies

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Fast rotation

Small Diameter (1-10km)

(vs 80-150 km)

Difficult to model due to small scale

http://www.photographyblogger.net/wpcontent/uploads/2009/05/ballerina6.jpg
http://home.earthlink.net/~dmac1137/sitebuildercontent/sitebuilderpictures/pirouette.jpg

Thank you!

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