


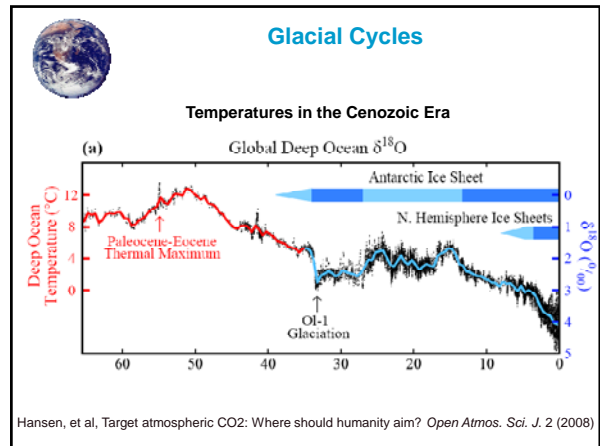
# Glacial Cycles

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Seminar on the Mathematics of Climate Change  
School of Mathematics  
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<http://www.tqmc.org/NYC052141/beginningpage.html>



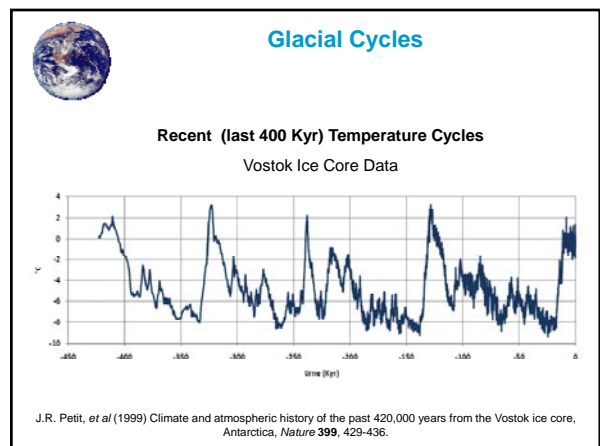
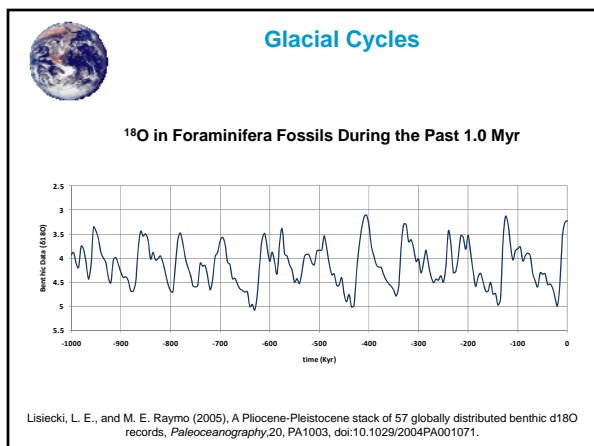
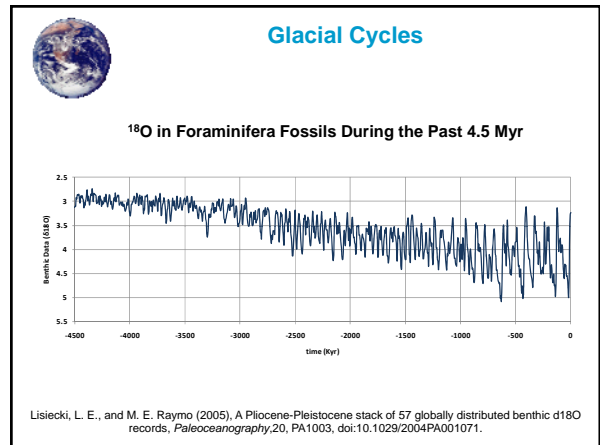
## Glacial Cycles

### <sup>18</sup>O as a Climate Proxy

The isotope <sup>18</sup>O preferentially evaporates from the ocean and is sequestered in glaciers, leaving the heavier isotope <sup>18</sup>O more highly concentrated in the ocean. Thus oceanic concentration of the isotope <sup>18</sup>O is higher during glacial periods.

Foraminifera absorb more <sup>18</sup>O into their skeletons when the water temperature is lower and when more <sup>18</sup>O is in the water.

Thus higher concentrations of <sup>18</sup>O in foraminifera fossils indicate lower ocean temperatures and higher glacier volume.





## Glacial Cycles

### What Causes Glacial Cycles?

#### Widely Accepted Hypothesis

The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

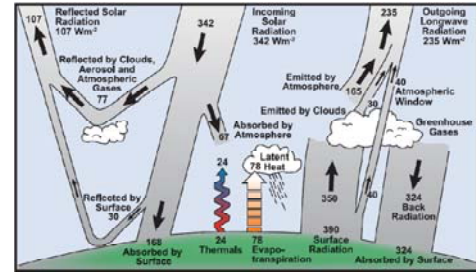
This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work are not fully understood.



## Glacial Cycles

### Heat Balance

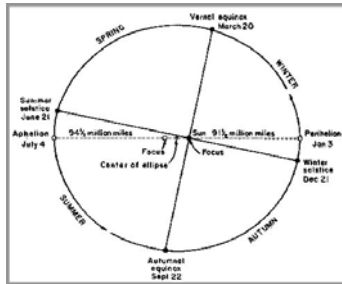


Historical Overview of Climate Change Science, IPCC AR4, p.96  
[http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1\\_Print\\_CH01.pdf](http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf)



## Glacial Cycles

### Eccentricity

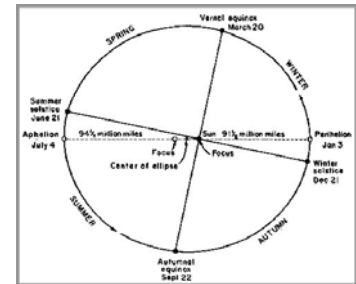


John Imbrie & Katherine Palmer Imbrie, *Ice Ages: Solving the Mystery*, Harvard Univ. Press, 1979.



## Glacial Cycles

### Eccentricity

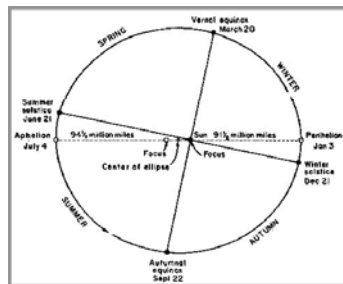


Perihelion:  $91.5 \times 10^6$  mi  
Aphelion:  $94.5 \times 10^6$  mi  
Semimajor axis:  $93 \times 10^6$  mi  
Eccentricity:  $1.5/93 = 0.016$



## Glacial Cycles

### Eccentricity



Perihelion: 91.5

Aphelion: 94.5

Change in radius:  
 $3/93 = 3.2\%$

Change in insolation: **6.4%**

Six percent less insolation  
in the southern winter than  
the northern winter.

6.4% of  $342 \text{ W/m}^2 =$

**$22 \text{ W/m}^2$**



## Glacial Cycles

### Global Annual Average Insolation

Solar output:  $K$  Watts

Solar intensity at distance  $r$  from the sun:

$$Q(t) = \frac{K}{4\pi r(t)^2} \text{ Wm}^{-2}$$

Cross section of Earth:  $\pi r_E^2 \text{ m}^2$

Global solar input:  $\frac{K r_E^2}{4r(t)^2} \text{ W}$

Total annual solar input ( $P =$  one year (in seconds)):

$$\int_0^P \frac{K r_E^2}{4r(t)^2} dt = \frac{K r_E^2}{4} \int_0^P \frac{dt}{r(t)^2} \text{ Joules}$$

**Glacial Cycles**  
Global Annual Average Insolation

Specific angular momentum (angular momentum per unit mass):  

$$\Omega = r^2 \dot{\theta} \quad \text{m}^2 \text{s}^{-1}$$

Total annual solar input:  

$$\frac{K r_E^2}{4} \int_0^P \frac{dt}{r(t)^2} = \frac{K r_E^2}{4} \int_0^P \frac{\dot{\theta} dt}{\Omega} = \frac{K r_E^2}{4\Omega} \int_0^{2\pi} d\theta = \frac{\pi K r_E^2}{2\Omega} \quad \text{Joules}$$

Mean annual solar input:  

$$\frac{\pi K r_E^2}{2P\Omega} \quad \text{Watts}$$

Mean annual solar intensity on the Earth's surface:  

$$\frac{\pi K r_E^2}{2P\Omega} \cdot \frac{1}{4\pi r_E^2} = \frac{K}{8P\Omega} \quad \text{Wm}^{-2}$$

**Glacial Cycles**  
Global Annual Average Insolation

Kepler's Third Law:  

$$P \sim a^{-3/2} \quad a = \text{semimajor axis}$$

Derived from Kepler:  

$$1 - e^2 \sim a\Omega^2 \quad e = \text{eccentricity}$$

Mean annual solar intensity:  

$$\frac{K}{8P\Omega} = \frac{\hat{K} a^{3/2} a^{3/2}}{\sqrt{1-e^2} \sqrt{1-e^2}} = \frac{\hat{K} a^2}{\sqrt{1-e^2}} \quad \text{Wm}^{-2}$$

**Glacial Cycles**  
Global Annual Average Insolation

Laskar:  

$$\frac{\hat{K} a^2}{\sqrt{1-e^2}}$$

**Fig. 11.** Variation of the semi-major axis of the Earth-Moon barycenter (in AU) from -250 to +250 Myr.

Semi major axis does not change much:  
 .005% corresponding to .01% change in global average insolation

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261-285.

**Glacial Cycles**  
Eccentricity

Note periods of about 100 Kyr and 400 Kyr.

The effect due to eccentricity is more significant, but not that much:  
 As  $e$  varies between 0 and 0.06,  $(1-e^2)^{-1/2}$  varies between 1 and 0.0018, or about 0.2%. (Twenty times the effect due to  $a$ .)

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261-285.

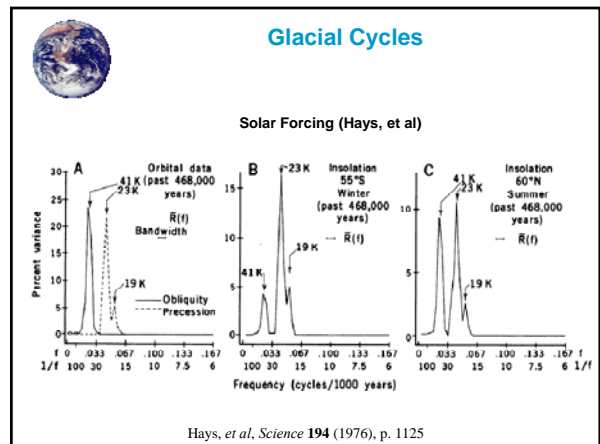
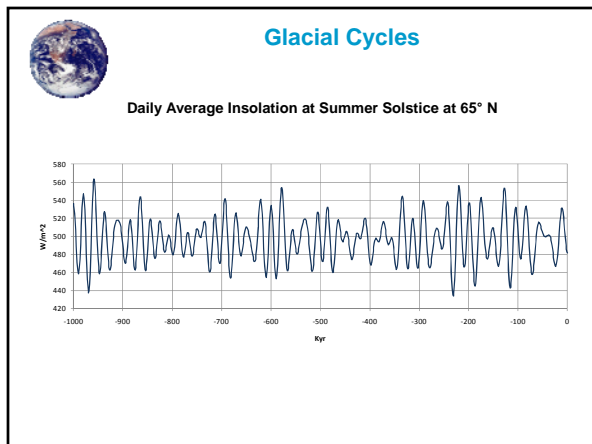
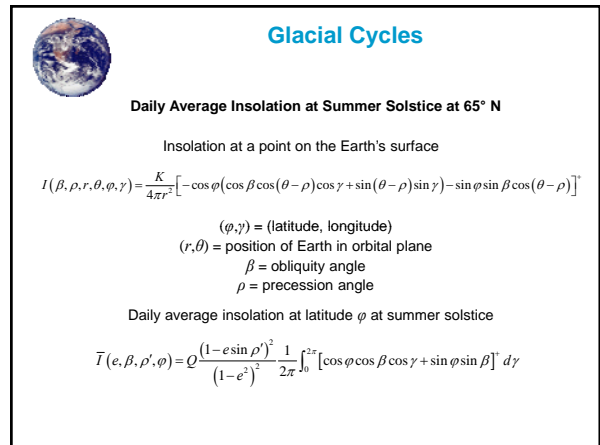
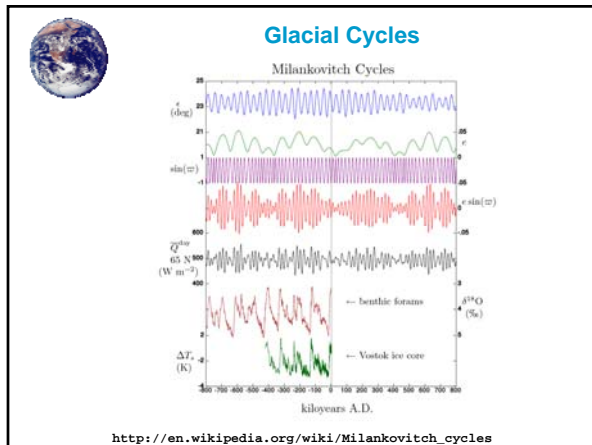
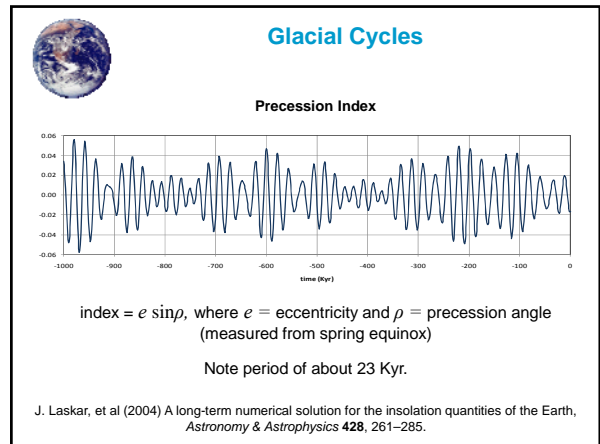
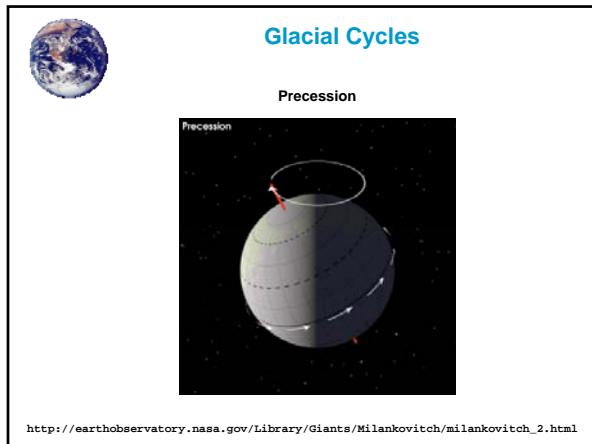
**Glacial Cycles**  
Obliquity

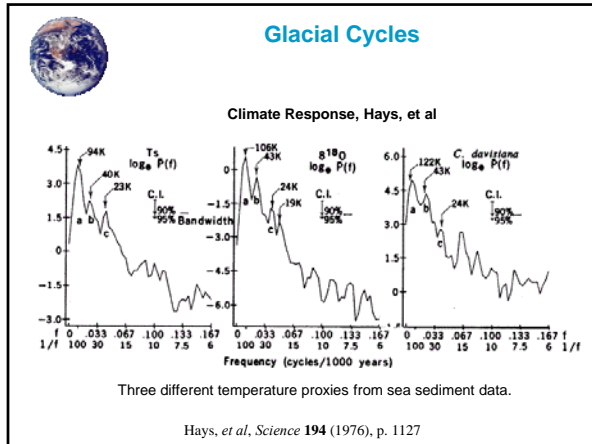
<http://upload.wikimedia.org/wikipedia/commons/6/61/AxialTiltobliquity.png>

**Glacial Cycles**  
Obliquity

Note period of about 41 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261-285.





### Glacial Cycles

Hays, et al, Summary

- 1) Three indices of global climate have been monitored in the record of the past 450,000 years in Southern Hemisphere ocean-floor sediments.
- 2) ... climatic variance of these records is concentrated in three discrete spectral peaks at periods of 23,000, 42,000, and approximately 100,000 years. These peaks correspond to the dominant periods of the earth's solar orbit, and contain respectively about 10, 25, and 50 percent of the climatic variance.

Hays, et al, *Science* 194 (1976), p. 1131

### Glacial Cycles

Hays, et al, Summary

- 3) The 42,000-year climatic component has the same period as variations in the obliquity of the earth's axis and retains a constant phase relationship with it.
- 4) The 23,000-year portion of the variance displays the same periods (about 23,000 and 19,000 years) as the quasiperiodic precession index.
- 5) The dominant, 100,000-year climatic component has an average period close to, and is in phase with, orbital eccentricity. Unlike the correlations between climate and the higher-frequency orbital variations (which can be explained on the assumption that the climate system responds linearly to orbital forcing), **an explanation of the correlation between climate and eccentricity probably requires an assumption of nonlinearity.**

Hays, et al, *Science* 194 (1976), p. 1131

### Glacial Cycles

Hays, et al, Summary

- 6) It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages.
- 7) A model of future climate based on the observed orbital-climate relationships, **but ignoring anthropogenic effects**, predicts that the long-term trend over the next seven thousand years is toward **extensive Northern Hemisphere glaciation\***.

\*Quoted by George Will, Washington Post, February 5, 2009

Hays, et al, *Science* 194 (1976), p. 1131

