Recent Developments in the Theory of Glacial Cycles

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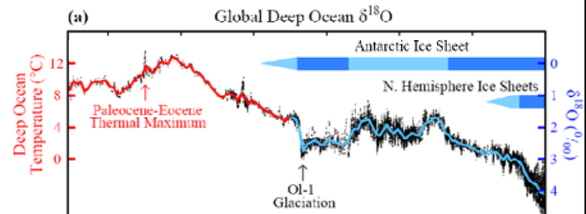
Seminar on the Mathematics of Climate Change
School of Mathematics
October 5, 2011

Glacial Cycles

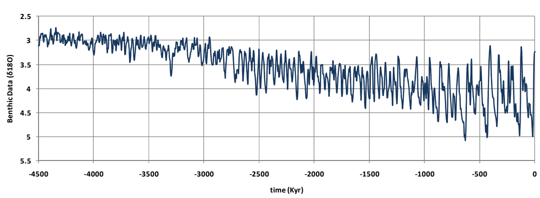
Temperatures in the Cenozoic Era



Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

Glacial Cycles

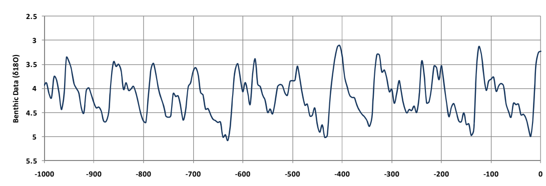
^{18}O in Foraminifera Fossils During the Past 4.5 Myr



Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

Glacial Cycles

^{18}O in Foraminifera Fossils During the Past 1.0 Myr

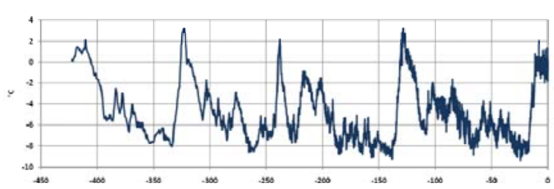


Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

Glacial Cycles

Recent (last 400 Kyr) Temperature Cycles

Vostok Ice Core Data



J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* **399**, 429-436.

Glacial Cycles

What Causes Glacial Cycles?

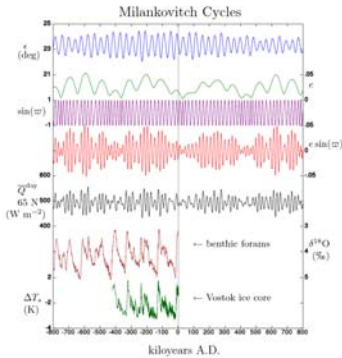
Widely Accepted Hypothesis

The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work is not fully understood.

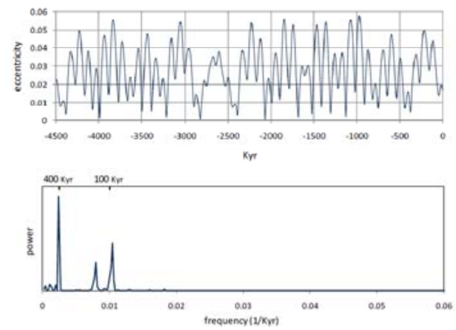
Glacial Cycles



http://en.wikipedia.org/wiki/Milankovitch_cycles

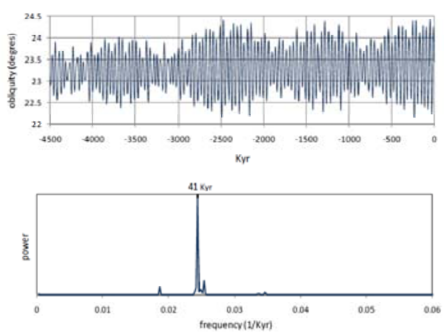
Glacial Cycles

Eccentricity



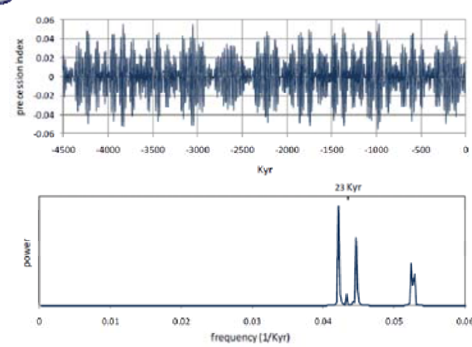
Glacial Cycles

Obliquity



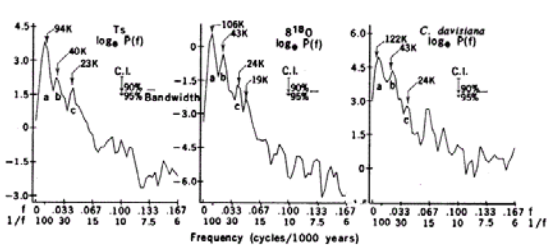
Glacial Cycles

Precession Index



Glacial Cycles

Climate Response, Hays, et al

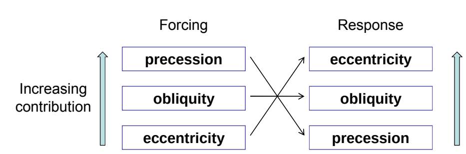


Three different temperature proxies from sea sediment data.

Hays, *et al*, *Science* **194** (1976), p. 1127

Glacial Cycles

Hays Summary



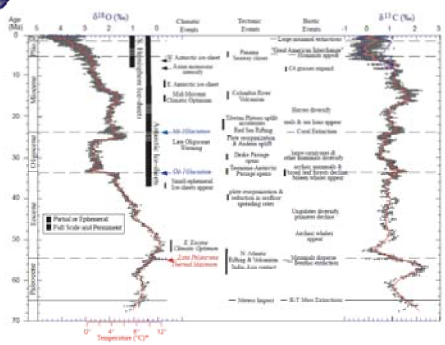
The explanation is that there are nonlinear feedbacks.

Is there another explanation?

Hays, *et al*, *Science* **194** (1976), p. 1127

Glacial Cycles

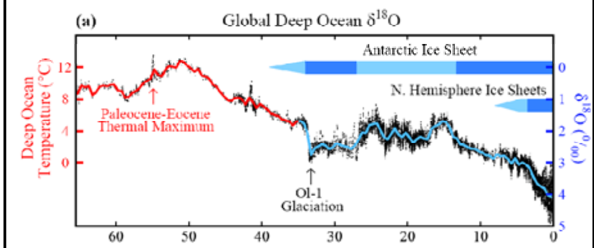
Cenozoic Era



Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Temperatures in the Cenozoic Era

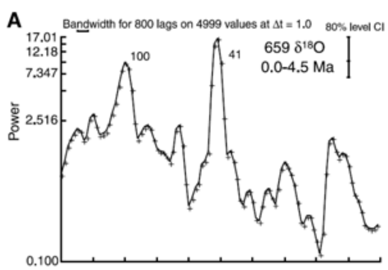


Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

Glacial Cycles

Climate Response (Zachos, et al)

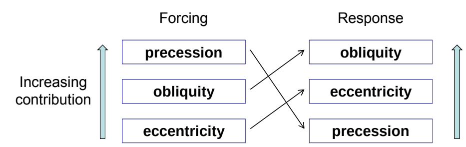
Power spectrum of climate for the last 4.5 Myr. Note the peaks at 41Kyr and 100 Kyr.



Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Zachos Summary



Forcing is defined as the maximum insolation at latitude 65° N.

Nonlinear effects?

Other explanations?

Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Why such a small precession contribution?

Incoming **So**lar Radi**ation** (*Insolation*), averaged over the entire globe and over a full year, depends only on eccentricity e, not on either obliquity or precession.

$$Q(e) = \frac{Q_0}{\sqrt{1-e^2}}$$

Insolation as a function of latitude, averaged over a full year, depends on eccentricity e and obliquity β, but not precession.

$$I = Q(e)\,s(\gamma,\beta)$$

where

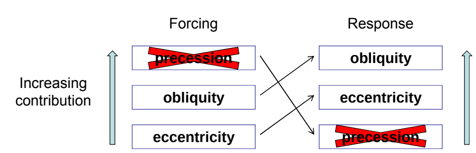
$$s(\theta,\beta) = \frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-(\cos\theta\sin\beta\cos\gamma - \sin\theta\cos\beta)^2}\,\cos\theta\, d\theta$$

θ = latitude

Glacial Cycles

Energy Balance

If we assume that glaciation depends on annual average insolation instead of insolation at summer solstice, then forcing and response are aligned.

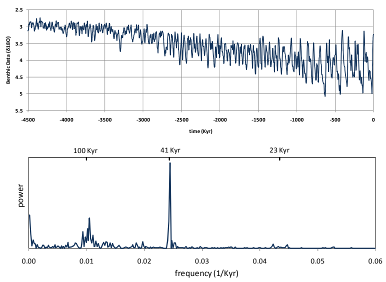


Glacial Cycles

18O in Foraminifera Fossils During the Past 4.5 Myr

data

power spectrum



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Glacial Cycles

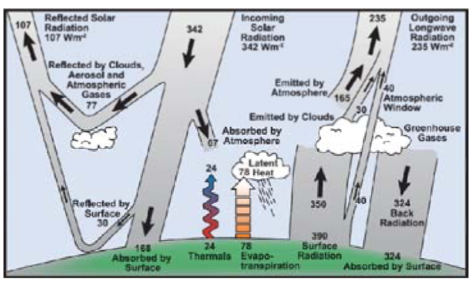
Why such a large obliquity contribution?

Possible explanation: **Ice-albedo feedback**

Ice reflects more energy than land or water.
more ice → less energy → colder → more ice
less ice → more energy → warmer → less ice

Glacial Cycles

Heat Balance



Historical Overview of Climate Change Science, IPCC AR4, p.96
`http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf`

Glacial Cycles

Budyko-Sellers Model

$$R\frac{\partial T}{\partial t} = \underbrace{Qs(y)}_{\text{insolation}}\underbrace{\left(1-\alpha(y,\eta)\right)}_{\text{albedo}} - \underbrace{(A+BT)}_{\text{re-radiation}} + \underbrace{C\left(\bar{T}-T\right)}_{\text{transport}}$$

$T = T(y,t) =$ annual mean surface temperature

$y = \sin(\text{latitude}) \quad y \in [0,1]$

$Q =$ global annual mean insolation

$s(y) =$ relative annual mean insolation, $\int_0^1 s(y)\,dy = 1$

$y = \eta:$ ice boundary

$$\alpha(y,\eta) = \begin{cases} \alpha_1, & y < \eta, \\ \alpha_2, & y > \eta. \end{cases} \quad \text{albedo}$$

$\bar{T}(t) = \int_0^1 T(y,t)\,dy =$ mean annual global temperature

K. K. Tung, *Topics in Mathematical Modeling*, Princeton University Press, 2007.

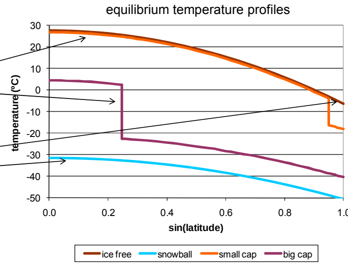
Glacial Cycles

Budyko-Sellers Model

Solve for equilibrium solution $T^*(y)$. Set right hand side = 0.

Two equilibrium solutions:
small cap: stable
large cap: unstable

Not equilibria:
ice free
snowball



Glacial Cycles

Budyko-Sellers Model

$$R\frac{\partial T}{\partial t} = Qs(y)\left(1-\alpha(y,\eta)\right) - (A+BT) + C\left(\bar{T}-T\right)$$

Note that the equilibrium solution $T^*(y)$ depends on Q and $s(y)$, which depend on the eccentricity e and the obliquity β. Therefore, the equilibrium location η of the ice boundary and the equilibrium global mean temperature (GMT) depend on the eccentricity and the obliquity.

We can use the computed values of eccentricity and obliquity to compute the ice boundary and GMT over the glacial cycles.

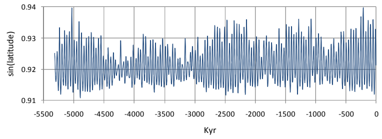
$$Q(e) = \frac{Q_0}{\sqrt{1-e^2}}$$

$$s(y,\beta) = \frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-\left(\sqrt{1-y^2}\sin\beta\cos\gamma - y\cos\beta\right)^2}\,d\gamma$$

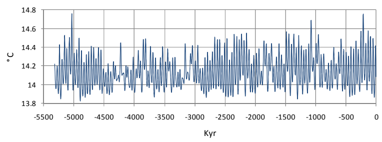
Glacial Cycles

Budyko-Sellers Model

ice boundary



global mean temperature

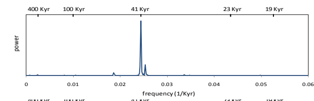


Glacial Cycles

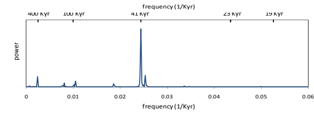
Budyko-Sellers Model

power spectra

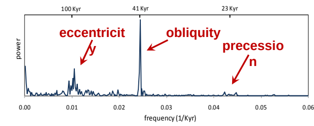
Model output: ice boundary



Model output: global mean temperature



Climate data

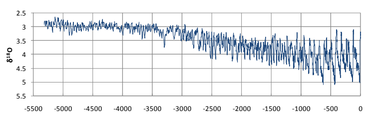


Glacial Cycles

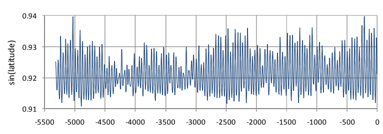
Budyko-Sellers Model

The ice-albedo feedback model correctly predicts the dominance of obliquity, but it fails to explain most of the other features of the climate data.

Climate data



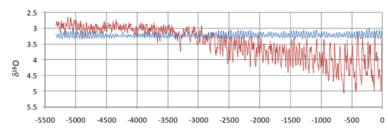
Model output

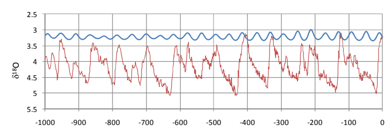


Glacial Cycles

Budyko-Sellers Model

red = data
blue = model



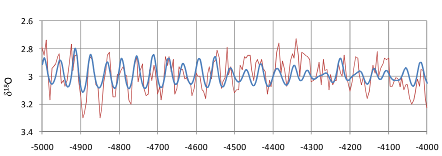


Glacial Cycles

Budyko-Sellers Model

Early Pliocene

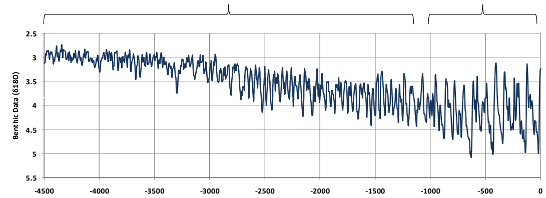
red = data
blue = model



Glacial Cycles

41 Kyr dominates

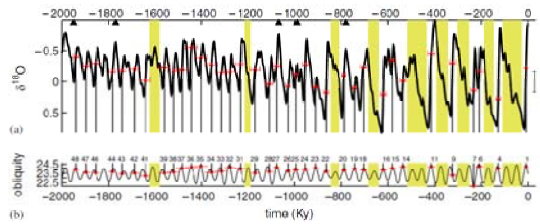
100 Kyr dominates



100,000 Year Problem: What's up with the last million years? Did eccentricity reassert itself? Or something else?

Glacial Cycles

Huyber's Analysis of Deglaciations



Red dots: deglaciations.

Peter Huybers, "Glacial variability over the last two million years: an extended depth-derived agemodel, continuous obliquity pacing, and the Pleistocene progression," *Quaternary Science Reviews* **26**, 37-55 (2007).

Glacial Cycles

Huyber's Analysis of Deglaciations

$$V_t = \begin{cases} V_{t-1} + \eta & \text{if } V_t < T_t \\ 0 & \text{if } V_t \geq T_t \end{cases}$$

$$T_t = at + b - c\theta'_t$$

V_t : ice volume at time t

T_t : threshold variable

η : rate of increase of ice volume

θ'_t : normalized obliquity

Units and constants

t : Kyr

V : chosen so that η = 1.

θ': mean zero and variance one

a = 0.05

b = 126

c = 20

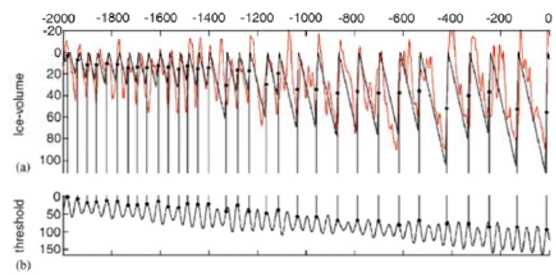
Glacial Cycles

Huyber's Analysis of Deglaciations

The deglaciations are triggered by obliquity cycles, but sometimes they don't trigger. When cycles are skipped, the deglaciations can be separated by 80 Kyr or 120 Kyr, creating the appearance of 100 Kyr cycles.

Glacial Cycles

Huyber's Analysis of Deglaciations



Glacial Cycles

Huyber's Analysis of Deglaciations

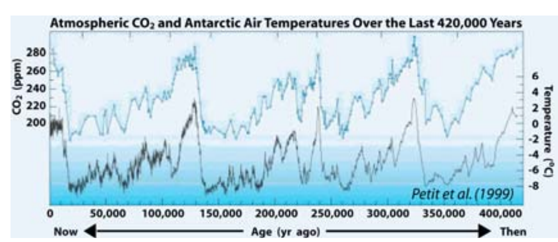
Huyber's model produces the decline in temperature and the increase in period and amplitude of the glacial cycles, but it depends heavily on an unspecified decline in the sensitivity of the triggering mechanism over last two million years.

What about greenhouse gases and the carbon cycle?

Andrew Hogg suggested a model incorporating the carbon cycle.

Glacial Cycles

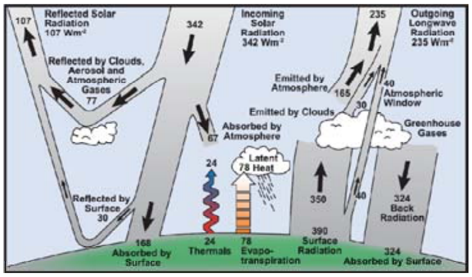
CO_2 as Feedback



Pam Martin, University of Chicago, 2010

Glacial Cycles

Heat Balance



Historical Overview of Climate Change Science, IPCC AR4, p.96
`http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf`

Glacial Cycles

Hogg's Model

$$c\frac{dT}{dt} = S(t) + G(C) - \sigma T^4,$$

$$\frac{dC}{dt} = V - \left(W_0 + W_1 C\right) + \beta\left(C_{\max} - C\right)\max\left(\frac{dT}{dt} - \varepsilon, 0\right).$$

weathering — CO2 outgassing — volcanos

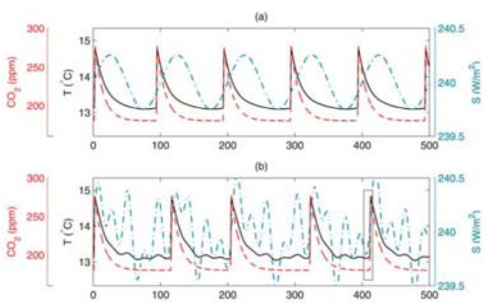
$$S(t) = \bar{S} + \sum_i S_i \sin\left(\frac{2\pi t}{\Gamma_i}\right)$$ insolation

$$G(C) = \bar{G} + A\ln\left(\frac{C}{C_0}\right)$$ greenhouse forcing

Andrew McC. Hogg, "Glacial cycles and carbon dioxide: A conceptual model," *Geophysical Research Letters* **35** (2008).

Glacial Cycles

Hogg's Model



Glacial Cycles

Hogg's Model

Hogg's model shows how the carbon cycle can act as a feedback amplifying and modifying the insolation forcing, but the forcing is somewhat artificial, and the triggering mechanism is difficult to justify. *Also, it does not solve the 100,000-year problem.*

What if the 100,000 year glacial cycle is not driven by eccentricity, but is a natural oscillation of the Earth's climate?

Saltzman and Maasch suggested just such a model.

Glacial Cycles

Salzman-Maasch Model

Milankovitch forcing

global ice mass $\longrightarrow$ $\dot{X} = -X - Y - uM(t)$

atmospheric CO_2 $\longrightarrow$ $\dot{Y} = -pZ + rY + sZ^2 - Z^2 Y$

deep ocean temperature $\longrightarrow$ $\dot{Z} = -q(X + Z)$

Barry Salzman and Kirk A. Maasch, "A Low-Order Dynamical Model of Global Climatic Variability Over the Full Pleistocene," *Journal of Geophysical Research* **95** (D2), 1955-1963 (1990)

Glacial Cycles

Salzman-Maasch Model

unforced

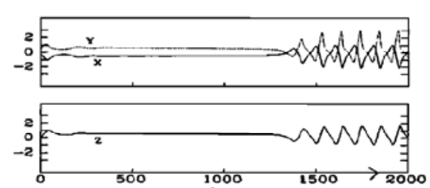


Fig. 7. Unforced (free) solution for (4)–(6) with $q = 1.2$, $s = 0.8$, and p and r varying linearly between $0.8 \rightarrow 1.0$ and $0.7 \rightarrow 0.8$, respectively. (Top) Model solutions for X (solid) and Y (dashed). (Bottom) Model solution for Z.

Glacial Cycles

Salzman-Maasch Model

forced

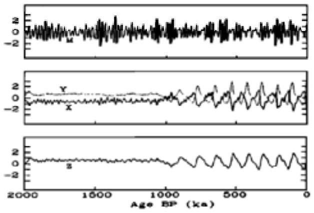


Fig. 8. Forced solution for (4)–(6) with $q = 1.2$, $s = 0.8$, $u = 0.7$, and p and r varying linearly between $0.8 \rightarrow 1.0$ and $0.7 \rightarrow 0.8$, respectively. (Top) Normalized 65°N July insolation curve (M) used as forcing. (Middle) Model solutions for X (solid) and Y (dashed). (Bottom) Model solution for Z.

Glacial Cycles

Salzman-Maasch Model

The Salzman-Maasch model shows how the carbon cycle and the ocean currents can interact to produce unforced oscillations with periods of about 100,000 years. The same model with slightly different parameters can exhibit stationary behavior. By forcing the model with Milankovitch cycles and by slowly varying the parameters over the last two million years, they can produce a bifurcation from small oscillations tracking the Milankovitch cycles to large oscillations with a dominant 100,000 year period.

Seems like a nice idea, but it is not widely accepted as the explanation.

Glacial Cycles

Questions

1. Did eccentricity play any role during the last million years, or is the apparent 100 Kyr cycle an artifact (Huybers).
2. Is the CO_2 feedback sufficient to explain the increasing amplitude and period of the glacial cycles during the last million years, *i.e.*, is it the mechanism behind the Huyber model.
3. Where does the atmospheric CO_2 go during the glacial maxima?
   The ocean? The land?
4. What will be the effect of the anthropogenic CO_2?