What is this? $\delta^{18}O$ (‰)

Example
Given a sample of calcium carbonate (CaCO$_3$) from a foraminifera fossil, suppose that the ratio of $^{18}$O atoms to $^{16}$O is $r = 0.002013$. How would we report this finding?

How would we measure it in the first place?
The instruments measure the difference between two samples. Typically, one measures the difference between the sample of interest and a standard sample. A common standard is something called "Vienna Standard Mean Ocean Water" (VSMOW), for which the ratio of $^{18}$O atoms to $^{16}$O is $s = 0.0020052$. Then

$$r = \frac{0.002013}{0.0020052} - 1 = 0.0007$$

So we would report

$\delta^{18}O = 3.9$ ‰

What is this? $\delta^{18}O$ (‰)

Going backwards, we have the formula

$$r = s(1 + \delta)$$

For example, if the sample is reported as $\delta^{18}O = 5$ ‰ using the VSMOW standard, then we translate to the ratio of $^{18}$O : $^{16}$O

$$r = 0.0020052(1 + 0.005) = 0.002015$$

$^{18}$O: 8 protons, 8 electrons, 10 neutrons
$^{17}$O: 8 protons, 8 electrons, 9 neutrons
$^{16}$O: 8 protons, 8 electrons, 8 neutrons

Most of the oxygen atoms on Earth are $^{16}$O. About 1 in 500 atoms is $^{18}$O. About 1 in 2500 is $^{17}$O.

There are other oxygen isotopes, but they are unstable.
Isotopes as Proxies

Common Standards

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>Ratio</th>
<th>Standard</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>D/H</td>
<td>0.0012558</td>
<td>VSMOW</td>
<td>Pierrehumbert¹</td>
</tr>
<tr>
<td>¹⁸O:¹⁶O</td>
<td>0.0112372</td>
<td>PDB</td>
<td>Wikipedia²</td>
</tr>
<tr>
<td>¹³C:¹²C</td>
<td>0.0020552</td>
<td>VSMOW</td>
<td>Pierrehumbert¹</td>
</tr>
<tr>
<td>²H:²H</td>
<td>0.0020674</td>
<td>VSMOW</td>
<td>Pierrehumbert¹</td>
</tr>
</tbody>
</table>

Standards:
VSMOW: Vienna Standard Mean Ocean Water
PDB: Pee Dee Belemnite
VPDB: Vienna Pee Dee Belemnite


Isotopes as Proxies

What does δ¹⁸O (%) tell us?

Fractionation

Example: Evaporation of Water

\[ \Delta \delta = \frac{\Delta \delta^* - 1}{2} = 1 + \frac{\Delta \delta^*}{2} - 1 = 1 + \frac{1}{2} \Delta \delta^* - 1 \]

Note that the standard drops out.

\[ f = \frac{1 + \Delta \delta^*}{2} \]

Since \( \Delta \delta^* \) and \( \delta^* \) are typically small, \( \Delta \delta^* \) is even smaller, so

\[ \Delta \delta = \delta - \Delta \delta^* \]

What does δD (%) tell us?

Example: Evaporation of Water

\[ \delta_D = \delta^* + \epsilon \]

0.002. Therefore, \( \epsilon = 0.002 \).

δD = δ - δD

Is there a better approximation for δD?

Summary

\[ \Delta \delta_D = \Delta \delta^* \delta \Delta \delta_D = 0.000822 \]

δD = ε = -0.01

Isotopes as Proxies

Example: Evaporation of Water

\[ \delta_D = \delta^* + \epsilon \]

Before

\[ \delta_D = \delta^* \delta \]

After

\[ \delta_D = \delta^* \delta^\prime \]

Suppose that 0.2% of the water becomes vapor, i.e., \( \delta = 0.002 \).

Summary

\[ \Delta \delta_D = \Delta \delta^* \delta \Delta \delta_D = 0.000822 \]

δD = ε = -0.01
What if all the glaciers melted? How would the deuterium content of seawater change?

\[ \delta D = \delta_0 \]

About 2% of the Earth’s water is in glaciers, vs. 98% in the oceans, so we take

\[ \delta = \delta_0 \]

According to Ray,

\[ \delta_0 = -420 \text{‰} \]

so

\[ \delta D = -8.4 \text{‰} \]

And it’s even more complicated.

\[ (\delta^{18}O) / T \approx -0.25 \text{‰} / ^\circ \text{C} \]

(Reference: Ray’s book)

And then there’s carbon.
Isotopes as Proxies

And then there's carbon.

\[ \text{photosynthesis} \]

\[ 6\text{CO}_2 + 6\text{H}_2\text{O} \rightarrow \text{C}_6\text{H}_{12}\text{O}_6 + 6\text{H}_2\text{O} \]

\[ \delta_1 = \delta^{13}\text{C} \]

\[ \delta_2 = \delta^{13}\text{C} \]

Fractionation is about -25‰.

\[ \delta_1 = \delta_2 - 0.025 \]

Result: Plants, animals, coal, and oil are all lighter in \(^{13}\text{C}\) than inorganic carbon.

**Isotopes as Proxies**


**Isotopes as Proxies**

*Biology Matters*


**Isotopes as Proxies**

*Paleocene-Eocene Thermal Maximum (PETM)*

Sharp decrease in \(^{18}\text{O}\), interpreted as a rapid increase in temperature.

Sharp decrease in \(^{13}\text{C}\), interpreted as massive oxidation of sequestered organic carbon.

**Isotopes as Proxies**

*Late Pleistocene Glacial Cycles*

Sharp decrease in \(^{18}\text{O}\), interpreted as a rapid glaciations.

Sharp increase in \(^{13}\text{C}\), interpreted as increase in biological activity.

Since the atmospheric \(\text{CO}_2\) increased during these periods, there must have been a large release of inorganic carbon, presumably from the oceans.

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**Isotopes as Proxies**

*What is this?*

depends more on temperature

depends more on ice volume

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**Isotopes as Proxies**

*Math and Climate Seminar IMA*

Mathematics and Climate Research Network

Joint MCRN/IMA Math and Climate Seminar

Tuesdays 11:15 – 12:05
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