Kicks and flows:
A dynamical systems approach to modeling resilience

Kate Meyer,
September 23, 2014

Kicks and flows:
A dynamical systems approach to modeling resilience

Outline

1. The resilience paradigm
2. Defining resilience mathematically
3. A kick-flow system
   a. Behavior in a linear system
   b. Experiments in 2D nonlinear systems
4. Future Directions
Resilience ... the capacity of [a] system to absorb change and disturbances and still retain its basic structure and function”  (p. 113)
1. The resilience paradigm

Resilience ... the capacity of [a] system to absorb change and disturbances and still retain its basic structure and function”

1. The resilience paradigm

Resilience ... the capacity of [a] system to absorb change and disturbances and still retain its basic structure and function”

2. Defining resilience mathematically

resilience of \textbf{WHAT} to \textbf{WHAT} ?

- perturbation of...
  - state variables
  - parameters

the basin of attraction

3. A kick-flow system
3. A kick-flow system

Linear 1D: Where is the attractor of this kick-flow system?

\[ x'(t) = -x \]

\[ V(x) = \frac{1}{2} x^2 \]

\[ x_e = \frac{k}{1 - e^{-\delta}} \]
3. A kick-flow system

Linear 2D

\[ X' = \begin{bmatrix} -7/4 & -7/4 \\ 5/4 & -5/4 \end{bmatrix} X \]
3. A kick-flow system

Linear 2D

\[
x' = -1.75x - 1.75y \\
y' = 1.25x - 1.25y
\]
3. A kick-flow system

Linear 2D

ODEs
initial position
kick size, directions
flow time

Plot of discrete system
3. A kick-flow system

Nonlinear 2D Predator-Prey model for species with limited growth

\[ x'(t) = x(3 - y - x) \]
\[ y'(t) = y(-0.5 + x - y) \]

http://math.rice.edu/~dfield/dfpp.html

(1.75, 1.25)
3. A kick-flow system

Nonlinear 2D

Predator-Prey model for species with limited growth

\[
x'(t) = x(3 - y - x) \\
y'(t) = y(-0.5 + x - y)
\]

kick size = 0
flow time = 0.1
iterations = 20
3. A kick-flow system

Nonlinear 2D Predator-Prey model for species with limited growth

\[ x'(t) = x(3 - y - x) \]
\[ y'(t) = y(-0.5 + x - y) \]

kick size = 0.1
kick directions = \( \pi \) to \( 3\pi/2 \)
flow time = 0.1
iterations = 80
3. A kick-flow system

Nonlinear 2D

\[ x'(t) = x(3 - y - x) \]
\[ y'(t) = y(-0.5 + x - y) \]

kick size = 0.1
kick directions = \( \pi \) to \( 3\pi/2 \)
flow time = 0.1
iterations = 80
4. Future Directions

• Kick-flow in interesting 2D systems
  o attracting periodic orbit
  o excitable
  o unstable linear

• Modify kick-flow
  o change potential
  o multiple kick types
  o move towards stochastic kicks

• Your ideas?
4. Future Directions

Kick-flow in 2D system with attracting periodic orbit

Hopf bifurcation normal form

\[ r' = r(1 - r^2) \]
\[ \theta' = 1 \]
4. Future Directions

\[ r' = r(1 - r^2) \]

\[ \theta' = 1 \]

kick size: 1
flow time: 1
4. Future Directions

\[ r' = r(1 - r^2) \]
\[ \theta' = 1 \]

kick size: 1
flow time: 1
4. Future Directions

\[ r' = r(1 - r^2) \]
\[ \theta' = 1 \]

kick size: 1
flow time: 1
4. Future Directions

\[ r' = r(1 - r^2) \]
\[ \theta' = 1 \]

kick size: 1
flow time: 1
4. Future Directions

\[ r' = r(1 - r^2) \]
\[ \theta' = 1 \]
4. Future Directions

\[ r' = r(1 - r^2) \]

\[ \theta' = 1 \]
### References


Details on kick-flow-equilibrium plot for 1d linear system

Calculation of $x_e(\delta, k)$

\[
x' = -x
\]

\[
\phi_t(x) = xe^{-t}
\]

want \quad \phi_\delta(x_e) = x_e - k,

so \quad x_e e^{-\delta} = x_e - k

\[
\implies x_e = \frac{k}{1 - e^{-\delta}}
\]

MATLAB commands (a=1):

\[
>> [D,K]=\text{meshgrid}(0.5 : 0.02 : 4, 0 : 0.02 : 3);
>> C=K./(1-exp(-D));
>> \text{surf}(D,K,C,'EdgeColor','none')
\]