Understanding Early Agricultural Impacts on Climate with Dynamical Systems

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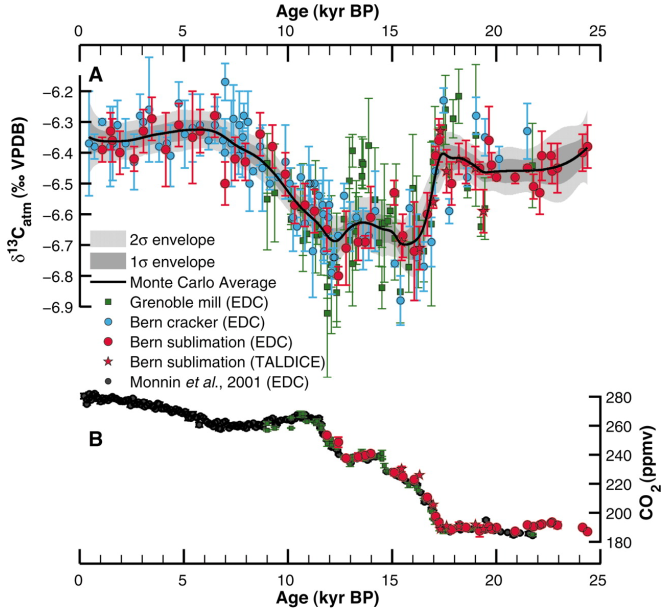
Motivation: The Ecological "Problem"

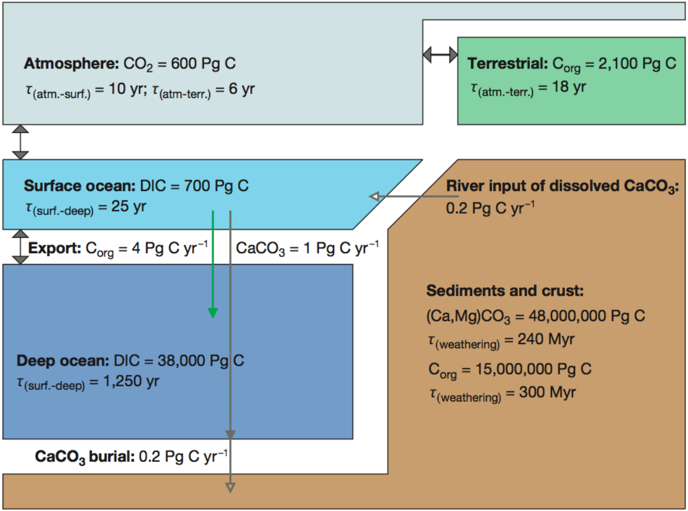


J. Schmitt, et. al. "Carbon Isotope Constraints on The Deglacial CO_2 Rise from Ice Cores." *Science.* (2012)

Outline

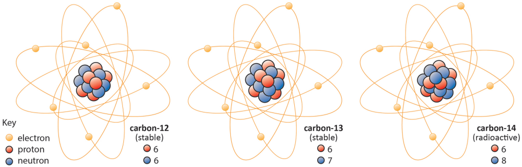
1. Background
   (i) The Carbon Cycle
   (ii) Carbon Isotopes
   (iii) Peatlands
2. Addressing the Problem
3. Continuing Work

Background: The Carbon Cycle



D. Sigman and E. Boyle. “Glacial/interglacial variations in atmospheric carbon dioxide.” *Nature*. (2000)

Background: Carbon Isotopes



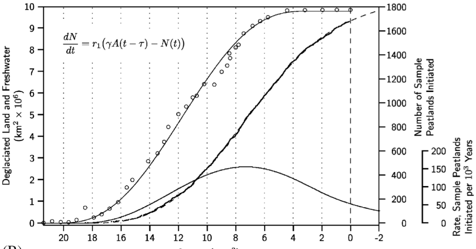
- Measure isotope ratio relative to a standard
  - $\delta^{13}\text{C} = \left(\frac{\left(\frac{^{13}C}{^{12}C}\right)_{sample}}{\left(\frac{^{13}C}{^{12}C}\right)_{PDB}} - 1 \right) \cdot 1000‰$
- Carbon's standard is PeeDee Belemnite
  - relatively high in ^{13}C
  - most δ^{13}C values are negative
- Different physical/biological processes prefer one isotope over the other

Background: What is peat?

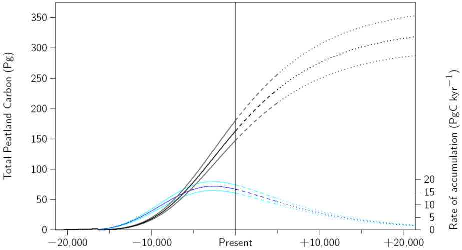



Background: Why do we care about peat?

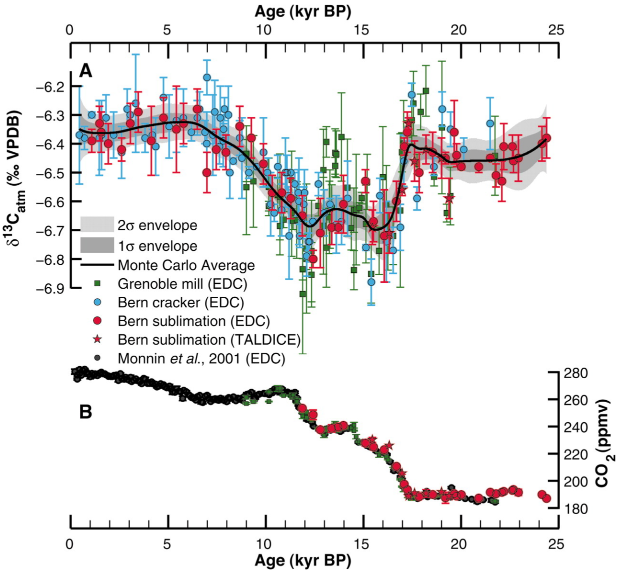
- $<3\%$ of total terrestrial land area[3]
- 1/3 of all carbon in terrestrial biosphere stored in peatlands[3]
- observed .096 PgC/year sequestration rate[3]



(B)

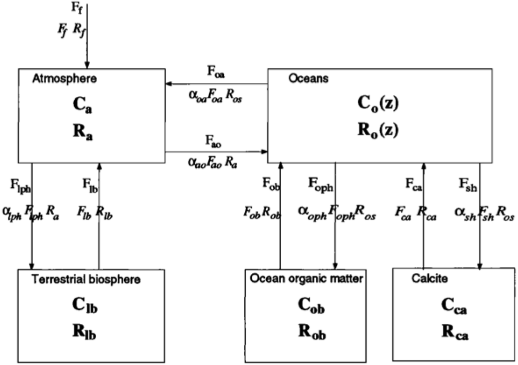


Quiz: Is the peatland data consistent with Schmitt et al.'s data?



J. Schmitt, et. al. "Carbon Isotope Constraints on The Deglacial CO_2 Rise from Ice Cores." *Science* (2012)

Approach #1: Use someone else's model



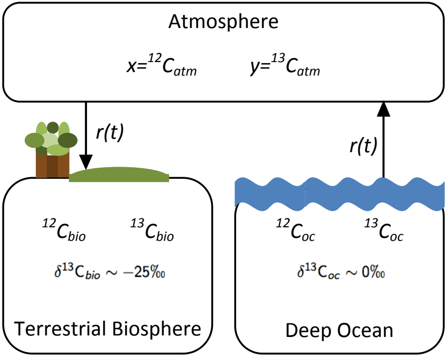
P. Tans et al. "Oceanic ^{13}C/^{12}C Observations." *Gl. Biogeochem. Cycles.* (1993)

- Assumptions about terrestrial biosphere, atmosphere, ocean organic matter, and calcite give the carbon content and isotopic ratio of the ocean for depth *z*
- Adjust assumptions to give us information about the atmosphere?

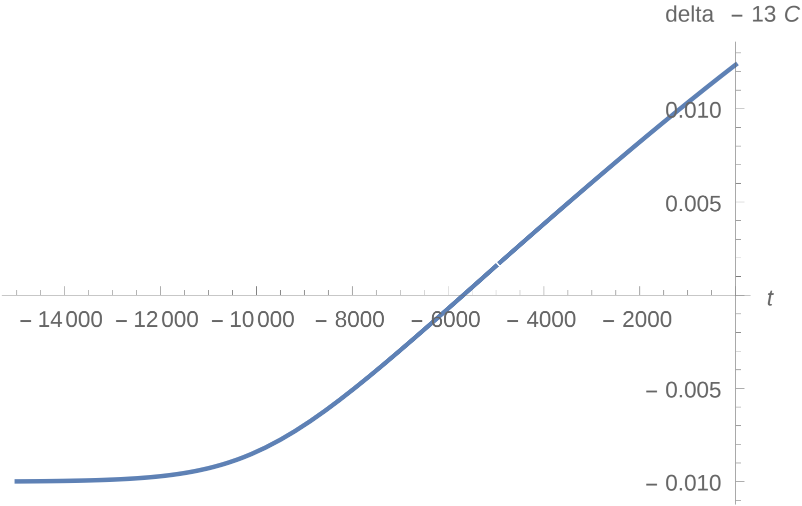
Approach #2: Simplified Box Model

$$\dot{x} = \left(\rho_{oc}^{x} - \rho_{bio}^{x}\right) r(t)$$
$$\dot{y} = \left(\rho_{oc}^{y} - \rho_{bio}^{y}\right) r(t)$$

$$\rho_i^x = \frac{1}{1 + \frac{^{13}C_i}{^{12}C_i}}, \quad \rho_i^y = \frac{\frac{^{13}C_i}{^{12}C_i}}{1 + \frac{^{13}C_i}{^{12}C_i}}$$



It's not so easy

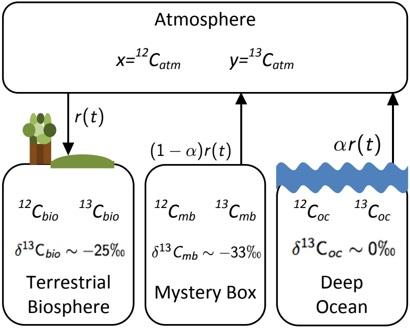


Approach #3: Something Mysterious

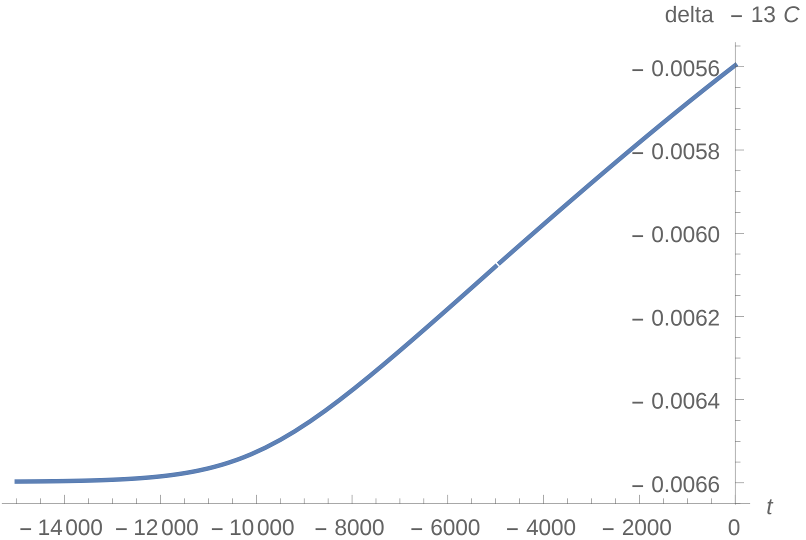
$$\dot{x}(t) = \left(\alpha\rho^x_{oc} + (1-\alpha)\rho^x_{mb} - \rho^x_{bio}\right) r(t)$$

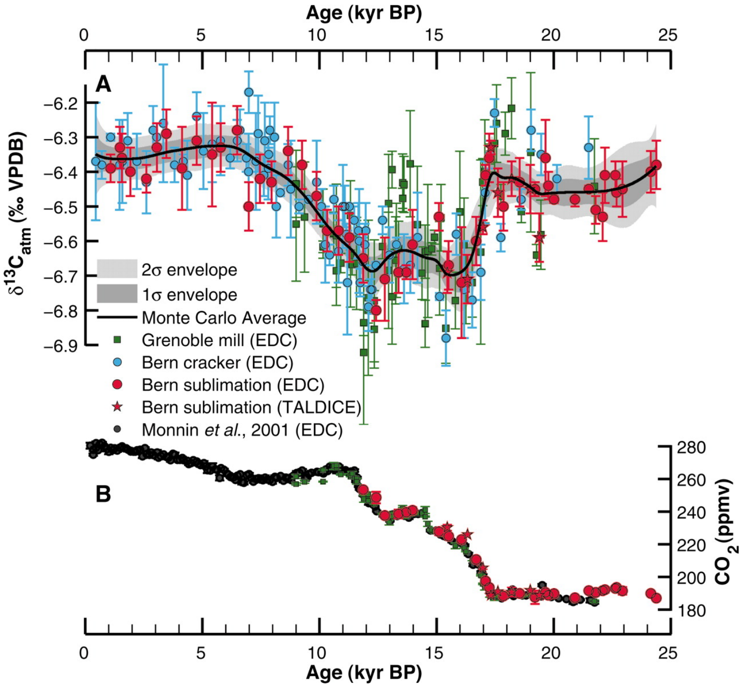
$$\dot{y}(t) = \left(\alpha\rho^y_{oc} + (1-\alpha)\rho^y_{mb} - \rho^y_{bio}\right) r(t)$$

$$\rho^x_i = \frac{1}{1+\frac{^{13}C_i}{^{12}C_i}}, \qquad \rho^y_i = \frac{\frac{^{13}C_i}{^{12}C_i}}{1+\frac{^{13}C_i}{^{12}C_i}}$$

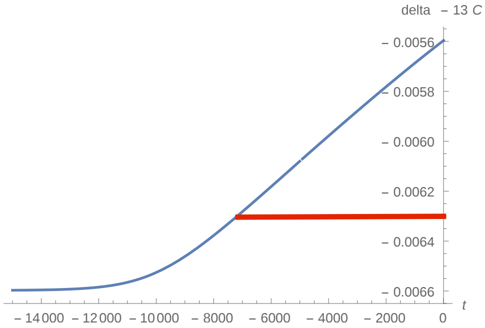


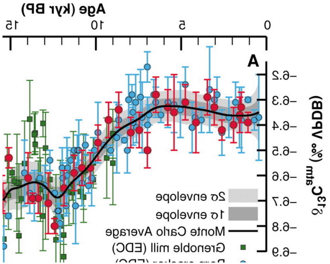
Results: $\alpha = .275$




Age (kyr BP)
0
5
10
15
20
25
A
−6.2
−6.3
−6.4
−6.5
−6.6
−6.7
−6.8
−6.9
δ13Catm (‰ VPDB)
2σ envelope
1σ envelope
Monte Carlo Average
Grenoble mill (EDC)
Bern cracker (EDC)
Bern sublimation (EDC)
Bern sublimation (TALDICE)
Monnin et al., 2001 (EDC)
B
280
260
240
220
200
180
CO2 (ppmv)
0
5
10
15
20
25
Age (kyr BP)

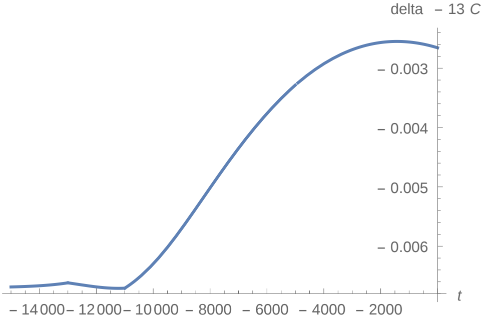
Results: $\alpha = .275$

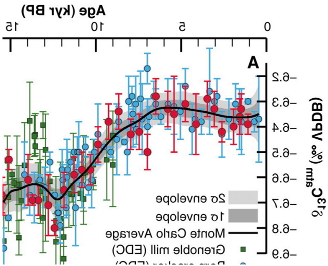




Results: α depends on t

α decreases from 1 to 0



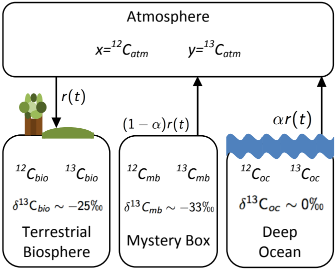


But atmospheric carbon isn't constant

$$\dot{x}(t) = (\alpha(t)\rho_{oc}^{x} + (1-\alpha(t))\rho_{mb}^{x} - \rho_{bio}^{x})\, r(t) + \frac{m(t)}{1 + c_1 + c_2\alpha(t)}$$

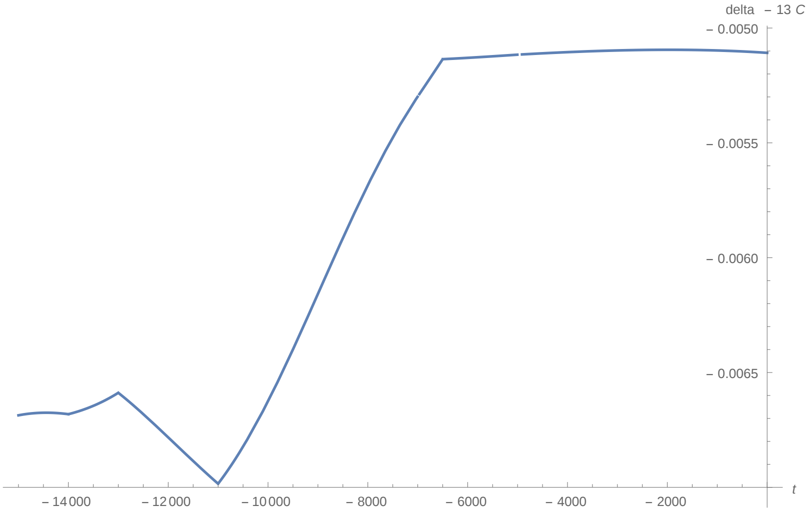
$$\dot{y}(t) = \left(\alpha(t)\rho_{oc}^{y} + (1-\alpha(t))\rho_{mb}^{y} - \rho_{bio}^{y}\right) r(t) + \frac{m(t)(c_1 + c_2\alpha(t))}{1 + c_1 + c_2\alpha(t)}$$

$$c_1 = \frac{{}^{13}C_{mb}}{{}^{12}C_{mb}}, \quad c_2 = \frac{{}^{13}C_{oc}}{{}^{12}C_{oc}} - \frac{{}^{13}C_{mb}}{{}^{12}C_{mb}}$$



Results: α and carbon depends on t

α decreases from 1 to .33 at -7ka then stays constant



Current Work

We *know* what's happening in the atmosphere

- Inverse problem to find α?
- Newton's method in function space

$$\alpha_{new} = \alpha_{guess} - (Df(\alpha_{guess}))^{-1} f(\alpha_{guess})$$

where $f(\alpha_{guess})$ is

$$f(\alpha) = \frac{y(t,\alpha)}{x(t,\alpha)} = \frac{y_0 + \int_{t_0}^{t} \dot{x}(s,\alpha)ds}{x_0 + \int_{t_0}^{t} \dot{y}(s,\alpha)ds}$$

References

1. J. Schmitt, et al. "Carbon isotope constraints on the deglacial CO2 rise from ice cores." *Science* **336**, 711-714 (2012)
2. D. Sigman and E. Boyle. "Glacial/interglacial variations in atmospheric carbon dioxide." *Nature* **407**, 859-869 (2000)
3. Eville Gorham et al. "Long-term carbon sequestration in North American peatlands." *Quat. Sci. Rev.* **58**, 77-82 (2012)
4. H. Stommel. "Thermohaline Convection with Two Stable Regimes of Flow." *Tellus* **13**, 224-230 (1961)
5. P. Tans et al. "Oceanic $^{13}C/^{12}C$ Observations: A New Window on Ocean CO_2 Uptake." *Global Biogeochemical Cycles* **7**, 353-368 (1993)