Perspectives on Resilience
Using Stommel’s Ocean-Box Model

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resilience:
the capacity of a system to absorb disturbance
and maintain its basic structure and function
state variable perturbations

parameter perturbations

Walker et al. (2004)
state variable perturbations

parameter perturbations

Walker et al. (2004)
\[ \phi : X \supseteq \]

**Definition:** \( A \) is an **attractor** for \( \phi \) if

1) \( A \) a nonempty, compact, invariant set, and

2) \( \exists \) a neighborhood \( U \) of \( A \) such that \( \omega(U) = A \)

(McGehee 1988)
Definition:

$B$ is an attractor block associated with $A$ if

1) $B$ is compact and nonempty
2) $\phi(B) \subset B^o$
3) $\omega(B) = A$

(McGehee 1988)
Define $\phi_\epsilon : \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ by

$$\phi_\epsilon(S) = \{ x \in X \mid \text{dist}(x, \phi(S)) < \epsilon \}$$
Let $P_\varepsilon(S)$ denote the set of all points accessible by $\varepsilon$-pseudo-orbits starting on $S$.
Definition: \( \beta(B) \equiv \sup\{ \epsilon \mid \phi_\epsilon(B) \subset B \} \)
**Definition:** For an attractor \( A \), the **intensity** of \( A \) is

\[
\nu(A) \equiv \sup\{\beta(B) \mid B \text{ is an attractor block associated with } A\}
\]

The **chain intensity** of \( A \) is

\[
\mu(A) \equiv \sup\{\epsilon \mid P_\epsilon(A) \subset \text{ compact set } \subset D(A)\}
\]

**Theorem:** \( \nu(A) \equiv \mu(A) \).

**Question:** Do these quantities measure “resilience”? 

(McGehee 1988)
Idea: Compute intensity of attraction in Stommel’s ocean box model

(Stommel 1961)
start point: a
map: time-1
(flow-kick) steps per trajectory: 50
# runs per plot: 50
Conclusion: When $\lambda = 0.2$,

$$\mu(a) \approx 0.7 \text{ and } \mu(c) \approx 0.4.$$ 

*Is $a$ more resilient than $c$?*
Parameter perturbation schedule:

Trajectories for different values of $T$: 
Critical time as a function of alternate $\lambda$ value
Smooth variation of $\lambda$:

$$\lambda = \frac{0.2}{1+\mu^2} + 0.2$$

$$\mu = ct$$
varying lambda smoothly from 0.2 to 0.4 and back

c = .12

c = .13

c = .14
Future directions:

- Does “intensity of attraction” have an analogue for vector fields?

- For a given schedule of parameter perturbations, can we predict analytically whether the system returns to its initial basin of attraction?

- How does resilience to state variables changes relate to resilience to parameter changes?
References


