The Earth's Energy Imbalance
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Energy Imbalance

Conservation of Energy

\[
\text{temperature change} - \text{energy in} = \text{energy out}
\]

short wave energy
long wave energy
from the Sun
from the Earth

Everything else is detail.

Energy Imbalance

Stefan-Boltzmann Law

\[
F = \sigma T^4
\]

power flux (W/m²)
temperature (K)

Stefan-Boltzmann constant
\[
\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4
\]

Example

temperature of the Sun: 5780 K
power flux: 5.67 x 10^4 \times (5780)^4 W/m²
where \( r_s \) = radius of the Sun = 6.96 x 10^8 m

200 nanoseconds = time it takes for the sun to produce the equivalent of the annual global electricity production (7.3 x 10^19 joules)

Insolation

Solar flux at a distance \( r \) from the sun:

\[
F = \frac{6.33 \times 10^7 \text{ Wm}^2}{4\pi r^2}
\]

\( r_s = 6.96 \times 10^8 \text{ m} \)
\( r = 1.5 \times 10^{10} \text{ m} \)
\( F = 1368 \text{ W/m}^2 \)

Power intercepted by the Earth:

\[
F = \sigma T^4 \text{ Watts}
\]

\( r_e = \text{radius of Earth} = 6.37 \times 10^6 \text{ m} \)
\( F = 1.74 \times 10^7 \text{ W} \)

Power intercepted by the Earth:

\[
F = \sigma r_e^2 T^4 \text{ W}
\]

Biologically Stored Energy

total coal reserves: 10^17 kg
energy content: 3 x 10^7 J/kg

2 days of Insolation
**Energy Imbalance**

**Insolation**

- Global Average Insolation
- Intercepted flux: \( F = 1368 \text{ W/m}^2 \)
- Earth cross-section: \( \pi r^2 \)
- Surface area: \( 4 \pi r^2 \)
- Average flux: \( 1368/4 = 342 \text{ W/m}^2 \)

Simple Model

Assume that Earth is a perfectly thermally conducting black body.

\[
Q = \sigma T^4
\]

\[
T = \left( \frac{Q}{\sigma} \right)^{1/4} = \left( \frac{342}{5.67 \times 10^{-8}} \right)^{1/4}
\]

\[
= 255K, -18 \text{ C} = 0 \text{ F}
\]

**Dynamics**

\[
\frac{dT}{dt} = -\frac{Q}{C_p} = \left(1 - \alpha \right) \sigma T^4
\]

stable equilibrium

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**Energy Imbalance**

**Albedo**

- Not all the insolation reaches the surface. Some is reflected back into space.
- The proportion reflected is called the albedo, denoted \( \alpha \).
- For Earth, \( \alpha \approx 0.3 \).

Simple Model

Assume that Earth is a perfectly thermally conducting black body, but only 70% of the insolation is absorbed.

\[
T = \left(0.7 \cdot \frac{Q}{\sigma} \right)^{1/4} = \left(0.7 \cdot \frac{342}{5.67 \times 10^{-8}} \right)^{1/4}
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**Energy Imbalance**

**Insolation vs. OLR**

- OLR = Outgoing Longwave Radiation

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**Energy Imbalance**

**Greenhouse Gases**

- Absorption Spectra
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Energy Imbalance

Assume all insolation goes toward warming.

Energy to melt all the sea ice: 1.3 W yr m⁻²
Power from Sun: 342 Wm⁻²
Time: 1.3/342 = 0.0038 yr = 33 hr

Assume a global heat imbalance of 0.85 W m⁻².

Energy to melt all the sea ice: 1.3 W yr m⁻²
Heat imbalance: 0.85 Wm⁻²
Time: 1.3/0.85 = 1.53 yr = 18 months

Assume all insolation goes toward warming.

Energy to raise sea level 1 meter by melting ice sheets: 9.3 W yr m⁻²
Power from Sun: 342 Wm⁻²
Time: 9.3/342 = 0.027 yr = 9.9 days

Assume a global heat imbalance of 0.85 W m⁻².

Energy to raise sea level 1 meter by melting ice sheets: 9.3 W yr m⁻²
Heat imbalance: 0.85 Wm⁻²
Time: 9.3/0.85 = 10.9 yr

Assume all insolation goes toward warming.

Energy to warm ocean 1°C to depth 1 km: 93 W yr m⁻²
Power from Sun: 342 Wm⁻²
Time: 93/342 = 0.27 yr = 3.3 months

Assume a global heat imbalance of 0.85 W m⁻².

Energy to warm ocean 1°C to depth 1 km: 93 W yr m⁻²
Heat imbalance: 0.85 Wm⁻²
Time: 93/0.85 = 109 years
Energy Imbalance

What if we melt all the ice sheets?
All ice sheets will raise the sea level 70 m. Will take 9.3 W yr m\(^{-2}\) \(\times\) 70 = 650 W yr m\(^{-2}\)

Assume all insolation goes toward warming.
Energy to melting all ice sheets: 650 W yr m\(^{-2}\)
Power from Sun: 342 Wm\(^{-2}\)
Time: 650/342 = 1.9 years
Assume a global heat imbalance of 0.85 W m\(^{-2}\).
Energy to melting all ice sheets: 650 W yr m\(^{-2}\)
Heat imbalance: 0.85 Wm\(^{-2}\)
Time: 650/0.85 = 765 years

 energy imbalance

Suppose that all the heat imbalance went to melting the glaciers.
It takes 9.3 Wyr/m\(^2\) to turn glaciers into 1 meter of ocean. If the heat imbalance is \(w\) W/m\(^2\), the sea level would rise at the rate of \(w/9.3\) meters per year. At the current imbalance of 0.85 W/m\(^2\), the rate is about 0.109 meters per year, or 1.09 meters per century.
Melting all the glaciers would cause a sea level rise of about 70 meters and would take about 765 years at the current imbalance.

Energy Imbalance

Suppose now that all the heat imbalance first goes to raising the top kilometer of ocean by 0.5 °C, and then goes to melting the glaciers.
It takes 46.5 Wyr/m\(^2\) to raise the temperature of a kilometer of ocean by 0.5 °C. If the heat imbalance is \(w\) W/m\(^2\), the increase would be achieved in 46.5/w years, after which the sea level would rise at \(w/9.3\) meters per year.
At the current imbalance of 0.85 W/m\(^2\), the ocean temperature increase would delay the sea level rise by about 55 years.

Energy Imbalance

Suppose instead that all the heat imbalance first goes to raising the top kilometer of ocean by 1 °C, and then goes to melting the glaciers.
It takes 93 Wyr/m\(^2\) to raise the temperature of a kilometer of ocean by 1 °C. If the heat imbalance is \(w\) W/m\(^2\), the increase would be achieved in 93/w years, after which the sea level would rise at \(w/9.3\) meters per year.
At the current imbalance of 0.85 W/m\(^2\), the ocean temperature increase would delay the sea level rise by about 109 years.

Energy Imbalance

Summary
Currently, it appears that the heat imbalance is mostly going to heating the ocean, not to melting ice. If this pattern continues, the danger for this century is more likely to come from weather changes than from sea level rise.
The current heat imbalance has the potential to raise the sea level by almost a meter per decade, a major threat to coastal cities worldwide.

Energy Imbalance

Questions about the Coming Centuries
How will the heat imbalance be divided between heating the ocean and melting the glaciers?
How will the heat imbalance be affected by increasing atmospheric greenhouse gases?
How will the heat imbalance be affected by increasing ocean temperatures?
What happens to the weather as the ocean temperature rises and the ice caps melt?
What should we do about coastal cities?
Energy Imbalance

What can mathematicians do?

Models
- Minimal complexity, aka "conceptual", "simple", "toy"
- Intermediate complexity
- Maximal complexity, aka "GCM"

Data
- Parameter estimation (statistics)
- Data assimilation

Quantification
- Uncertainty
- Resilience

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