Challenges in Adapting the Budyko Energy Balance Model to Pluto

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Budyko’s Energy Balance Model

\[ R \frac{\partial T}{\partial t} = Q_s(y)(1 - \alpha(y, \mu)) - (A + BT) + C(\bar{T} - T) \]
Budyko’s Energy Balance Model for Pluto?

\[ R \frac{\partial T}{\partial t} = \ldots ? \]
Some Particularly Challenging Challenges

- Determining appropriate parameter values
  - Two parameters may be especially difficult to determine

- Understanding “ice line” dynamics
  - What kind of ice are we talking about?
  - How should the ice lines be oriented?
Budyko’s Equation

“change in temperature over time”

\[ \frac{\partial T}{\partial t} = Qs_\beta(y)(1 - \alpha(y, \mu)) - (A + BT) + C(\bar{T} - T) \]

- Change in temperature by sine of latitude \((y)\)
- \(R\) is the Earth’s heat capacity
Budyko’s Equation

“change in temperature over time”

\[ \frac{\partial T}{\partial t} = \text{“energy in”} - \text{“energy out”} + \text{“energy transfer”} \]

\[ R \frac{\partial T}{\partial t} = Qs_\beta(y)(1 - \alpha(y, \mu)) - (A + BT) + C(\bar{T} - T) \]

- **Incoming solar radiation**
- **Q** is the integrated total solar input into the system
- Distribution of insolation \((s_\beta)\) depends on sine of latitude
- Albedo \((\alpha)\) dependent on both sine of latitude and the location of the ice lines \((\mu = (\mu_N, \mu_S))\)
Budyko’s Equation

“change in temperature over time”

\[
R \frac{\partial T}{\partial t} = Qs_\beta(y)(1 - \alpha(y, \mu)) - (A + BT) + C(\overline{T} - T)
\]
Budyko’s Equation

“change in temperature over time”

= “energy in” − “energy out” + “energy transfer”

\[ R \frac{\partial T}{\partial t} = Qs_\beta(y)(1 - \alpha(y, \mu)) - (A + BT) + C(\bar{T} - T) \]

▶ Outgoing radiation
▶ Linear approximation of Stefan-Boltzmann black body radiation
Budyko’s Equation

“change in temperature over time”

= “energy in” − “energy out” + “energy transfer”

\[ R \frac{\partial T}{\partial t} = Q_s \beta(y)(1 - \alpha(y, \mu)) - (A + BT) + C(\overline{T} - T) \]

► Zonal energy transfer
► \( C = 1.6 \) \( B \)
► \( \overline{T} \) is global average temperature, dependent on the ice lines \( \mu \)
Determining Parameter Values

\[ R \frac{\partial T}{\partial t} = Q_s \beta(y)(1 - \alpha(y, \mu)) - (A + BT) + C(T - \bar{T}) \]

- In the Earth model, \( A \) and \( B \) are determined empirically from satellite data
- We will use the Stefan-Boltzmann law:

\[ I(y) = \delta \sigma \ T^4 \approx \delta \sigma \ T_0^4 \left(1 + \frac{4(T - T_0)}{T_0} \right) \]

\[ A = \delta \sigma \ T_0^4, \quad B = \frac{4A}{T_0} \]
Determining Parameter Values

\[ A = \delta \sigma T_0^4, \quad B = \frac{4A}{T_0} \]

- \( 0 \leq \delta \leq 1 \) is determined by composition of surface and atmosphere
- Pluto has an atmosphere \ldots right now

- \( \delta \) may change drastically over the course of a Pluto year [Stansberry and Yelle]
Determining Parameter Values

▶ Option #1:
  ▶ Take $\delta = .5$
  ▶ Take $T_0 = 36^\circ K$ so that the temperature profile is in an appropriate range

▶ Option #2:
  ▶ Take $T_0 = 37.5^\circ K$, Pluto’s black body temperature
  ▶ Take $\delta = .45$ so that the temperature profile is in an appropriate range
Nitrogen ice is the most abundant ice on Pluto’s surface by a factor of 50 [Owen]

Darker regions on Pluto’s surface are composed of tholins [Betz]
Ice Line Dynamics: The Albedo Function

\[
\alpha(y, \mu) = \begin{cases} 
\alpha_1 & y < \mu_S \\
\alpha_2 & \mu_S < y < \mu_N \\
\alpha_1 & y > \mu_N 
\end{cases}
\]

- Nitrogen ice has albedo of .8 [Stransberry and Yelle]
- Tholins have albedos ranging from .01 to .15 [Cruikshank]
At Earth’s surface pressure, 1 atm, nitrogen freezes at 63.15 °K.

Pluto’s surface pressure is $10^{-6}$ atm and average temperature is 44 °K.

Nitrogen’s phase properties are not well understood at such low temperatures and pressures.

Satorre et al. give a freezing temperature of 22 °K at $10^{-10}$ atm.
Equilibrium Solutions

Equilibrium solutions, $T^*$, will satisfy

$$0 = Qs_\beta(y)(1 - \alpha(y, \mu)) - (A + BT^*(y, \mu)) + C(\bar{T}^*(\mu) - T^*(y, \mu)).$$

To find $T^*$, first solve for $\bar{T}^*$ by integrating in $y$.

The resulting equilibrium temperature profile is

$$T^*(y, \mu) = \frac{1}{B + C}(Qs_\beta(y)(1 - \alpha(y, \mu)) - A + C \bar{T}^*(\mu)).$$
Equilibrium Solutions

\[ \alpha(y, \mu) = \begin{cases} 
\alpha_1 & y < \mu_S \\
\alpha_2 & \mu_S < y < \mu_N \\
\alpha_1 & y > \mu_N 
\end{cases} \]

The lefthand and righthand limits at the southern ice line are

\[ T^*(\mu_{S-}, \mu) = \frac{1}{B + C}(Qs_\beta(\mu_S)(1 - \alpha_1) - A + C \bar{T}^*(\mu)) \]

\[ T^*(\mu_{S+}, \mu) = \frac{1}{B + C}(Qs_\beta(\mu_S)(1 - \alpha_2) - A + C \bar{T}^*(\mu)) \]

At equilibrium

\[ T_c = \frac{T^*(\mu_{S-}, \mu) + T^*(\mu_{S+}, \mu)}{2} = T^*(\mu_S, \mu) \]
Dynamics of the Ice Line

\[ \begin{bmatrix} \dot{\mu}_S \\ \dot{\mu}_N \end{bmatrix} = \begin{bmatrix} T^*(\mu_S, \mu) - T_c \\ T_c - T^*(\mu_N, \mu) \end{bmatrix} \]

\[ \begin{bmatrix} \dot{\mu}_S \\ \dot{\mu}_N \end{bmatrix} = \begin{bmatrix} T_c - T^*(\mu_S, \mu) \\ T^*(\mu_N, \mu) - T_c \end{bmatrix} \]
Vector Field with Equilibria, $T_c = 25 \degree K$
Vector Field with Equilibria, $T_c = 30^\circ K$
Vector Field with Equilibria, $T_c = 35 \, ^\circ\text{K}$
Vector Field with Equilibria, $T_c = 40 \, ^\circ K$
Vector Field with Equilibria, $T_c = 45^\circ K$
Vector Field with Equilibria, $T_c = 50 \, ^\circ K$
Take Aways

- There is a lot of uncertainty in some parameter values
  - but we will know more when New Horizons sends its information back

- Pluto’s simplified “ice lines” are unstable
  - but we haven’t accounted for the fact that Pluto may lose its atmosphere in part of its orbit
Looking Forward

- Define a function space and study the dynamics
  - McGehee and Widiasih used a four dimensional function space, but they used a quadratic approximation for insolation
  - We need at least a sixth order Legendre polynomial to capture the proper insolation distribution for Pluto

- Allow for four ice lines
  - Preliminary work with the Budyko model on Earth with four ice lines yield interesting results
  - Could we get stable solutions on Pluto with this configuration?
References


