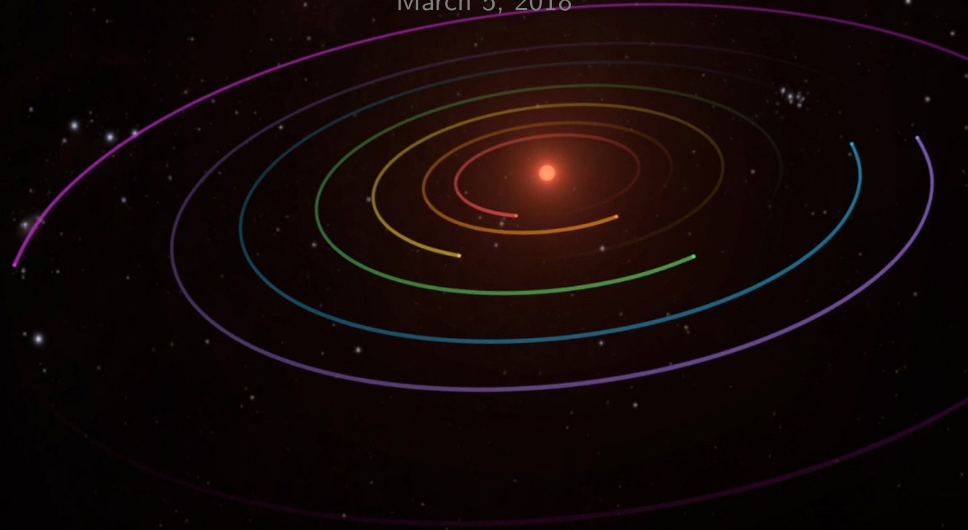
Effects of Eccentricity on Climate and Habitability

Harini Chandramouli

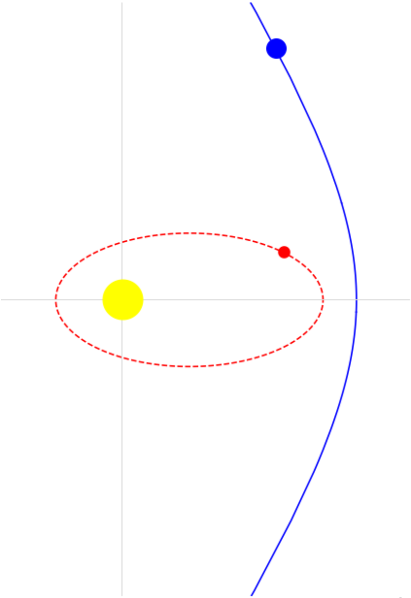
March 5, 2018



NASA/JPL-Caltech

The Scattering Problem

- Star passing “close” to our solar system



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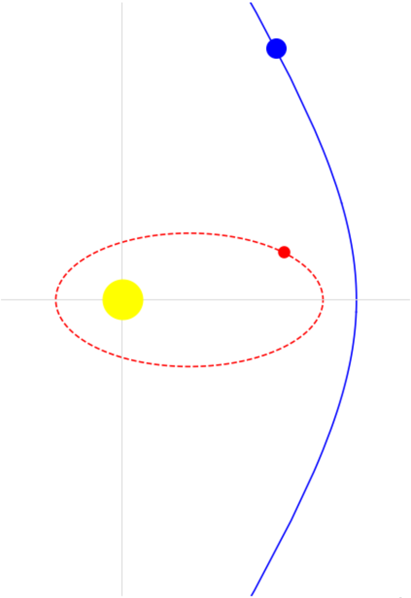
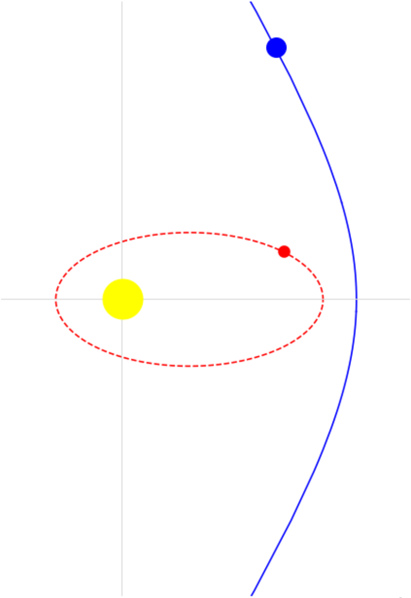
- Star passing “close” to our solar system
- Hyperbolic Restricted 3-Body Problem



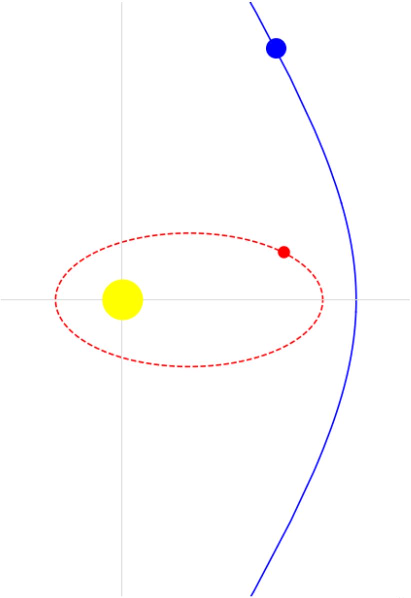
The Scattering Problem

- Star passing "close" to our solar system
- Hyperbolic Restricted 3-Body Problem
- Changes in some orbital elements $\implies$ changes in climate
  - Eccentricity
  - Obliquity
  - Precession



Logistics of the HR3BP

- Move around in the plane – let Sun be at origin



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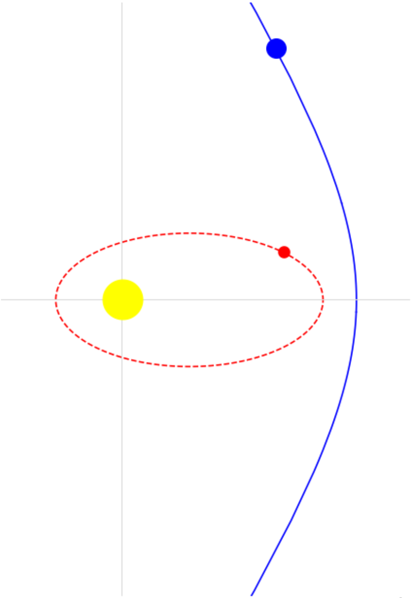

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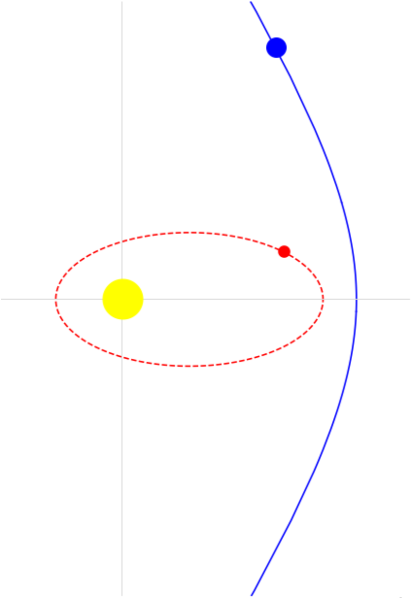

Logistics of the HR3BP

- Move around in the plane – let Sun be at origin
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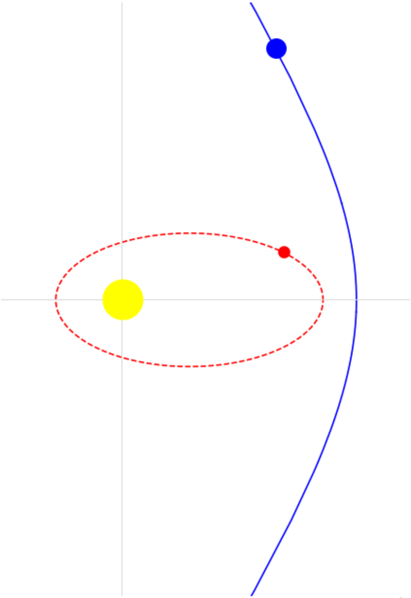
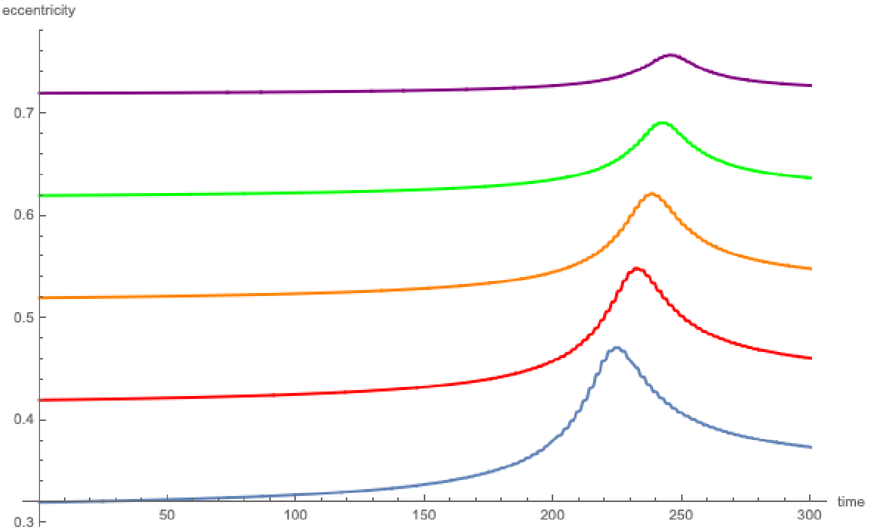
- 2D
- Formula for eccentricity vector

$$\mathbf{e} = \left(\frac{q_1}{q} - \frac{c}{\mu} p_2 \, , \, \frac{q_2}{q} + \frac{c}{\mu} p_1 \right)$$

  - (q_1, q_2) – position
  - $q = \sqrt{q_1^2 + q_2^2}$
  - c – angular momentum
  - μ – “mass”
  - (p_1, p_2) – momentum



Changes in Eccentricity in 2D Case



Change in eccentricity over time due to the passing star. Light Blue – $m_1 = m_2 = 0.5$, Red – $m_1 = 0.6$, $m_2 = 0.4$, Orange – $m_1 = 0.7$, $m_2 = 0.3$, Green – $m_1 = 0.8$, $m_2 = 0.2$, Purple – $m_1 = 0.9$, $m_2 = 0.1$.

Questions...

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  Insolation!

The Budyko Energy Balance Model

$$R\frac{\partial T}{\partial t} = \underbrace{Q\,s(y)\,(1-\alpha)}_{\text{incoming radiation}} - \underbrace{(A + B\,T(y,t))}_{\text{OLR}} - \underbrace{C\left(T(y,t) - \overline{T}(t)\right)}_{\text{heat transport*}}$$

- $T(y,t)$ – surface temperature at time t at latitude $\arcsin(y)$
- R – heat capacity of the surface of the planet
- Q – annual radiation from the Sun
- $s(y)$ – latitudinal distribution of energy
- $(1-\alpha)$ – fraction of radiative energy absorbed by the planet
- A, B, C – empirical parameters
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The Budyko-Widiasih Model

Add in the dynamic ice line equation:

$$\frac{d\eta}{dt} = \varepsilon \left(T_c - T(\eta) \right)$$

- η — ice line location
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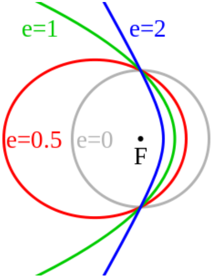
Eccentricity



Wikipedia

The main component of Earth's orbit affecting the global annual **in**coming **sol**ar radi**ation** (insolation), $Qs(y)$, averaged over the entire surface of Earth over an entire year, is the eccentricity of the ellipse that defines the orbit of Earth.

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  Exoplanets around M-Dwarf Stars

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phys.org

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  - Range in mass from about 0.075 to 0.50 solar mass
  - Surface temperature $< 4,000$ K
  - Most common type of star in the neighborhood of the Sun in the Milky Way
- This zone is so close to the star that gravitational tides are expected to lock a planet into spin-orbit resonance states.

Spin-Orbit Resonances

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Ex. Mercury rotates three times during every two orbits, so $p = 3/2$.

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The outer edge of the **habitable zone** is defined as the furthest distance at which liquid water on a planetary surface is not completely frozen.

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- $0 \leq e \leq 0.934$
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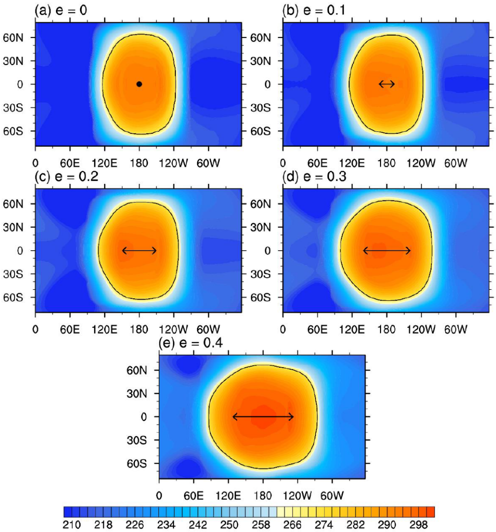
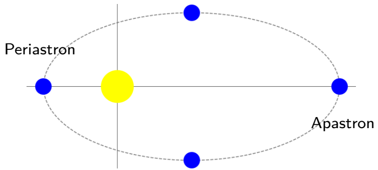
According to [6], when $p = 1$, we have the most stable climate and the widest habitable zone and eccentricity shrinks the width of the habitable zone.

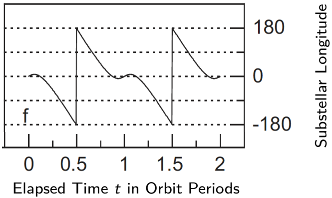
$p = 1$ Spin-Orbit Resonance State, [6]

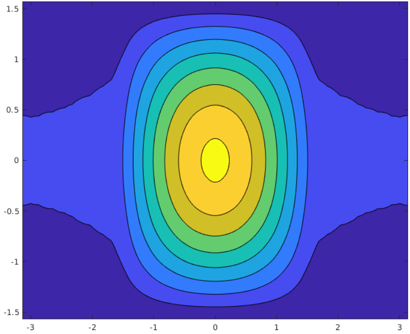


Long-term mean surface temperatures for different eccentricities. Units are K. The black arrow shows the migrations of the substellar points during an orbital cycle.

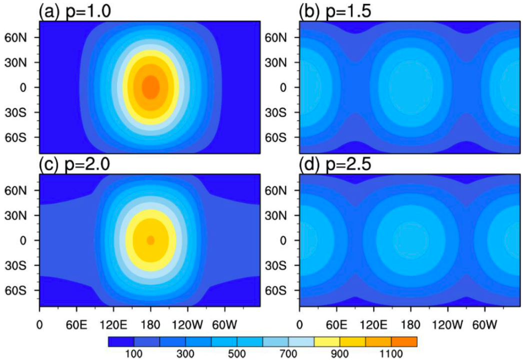
$p = 2$ Spin-Orbit Resonance, $e = 0.4$





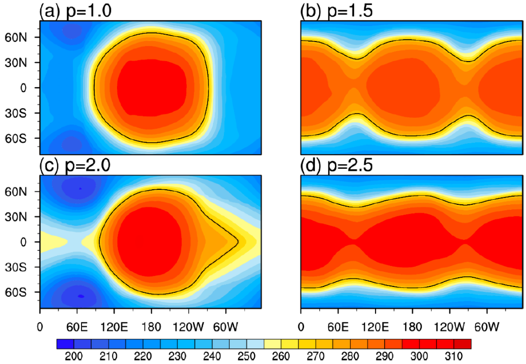


Insolation for Other Spin-Orbit Resonance States ($e = 0.4$), [6]



Incident stellar flux at the top of the atmosphere averaged over one orbital cycle for different spin-orbit resonance states. The contour interval is 100 Wm^{-2}.

Temperature for Other Spin-Orbit Resonance States ($e = 0.4$), [6]



Annual mean surface temperature averaged over one orbital cycle for different spin-orbit resonance states. Black curve shows the sea ice boundaries.

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- Harder to get $p = 1.0, 2.0$ to transition into snowball state, requires a greater decrease in insolation

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- Can we adapt the Budyko-Sellers Model to take into account the spin-orbit resonances and eccentricity differences on these exoplanets?

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- Can we adapt the Budyko-Sellers Model to take into account the spin-orbit resonances and eccentricity differences on these exoplanets?
  Well, that's what we're working on!

Nondimensionalizing the Budyko Model

Non-dimensional constants:

$$\delta = \frac{A + BT_c}{Q}$$
$$\gamma = \frac{C}{B}$$

This yields the nondimensional Budyko equation

$$\frac{\partial \varphi}{\partial \tilde{t}} = s(\tilde{y})\left(1 - \alpha(\tilde{y}, \tilde{t})\right) - \delta - \varphi(\tilde{y}, \tilde{t}) - \gamma\left(\varphi(\tilde{y}, \tilde{t}) - \overline{\varphi}(\tilde{t})\right)$$

Budyko Model with Eccentricity as a Parameter

$$Q \to Q(e) = \frac{Q_0}{\sqrt{1 - e^2}}$$

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Here...

- ν = true anomaly
- β = obliquity

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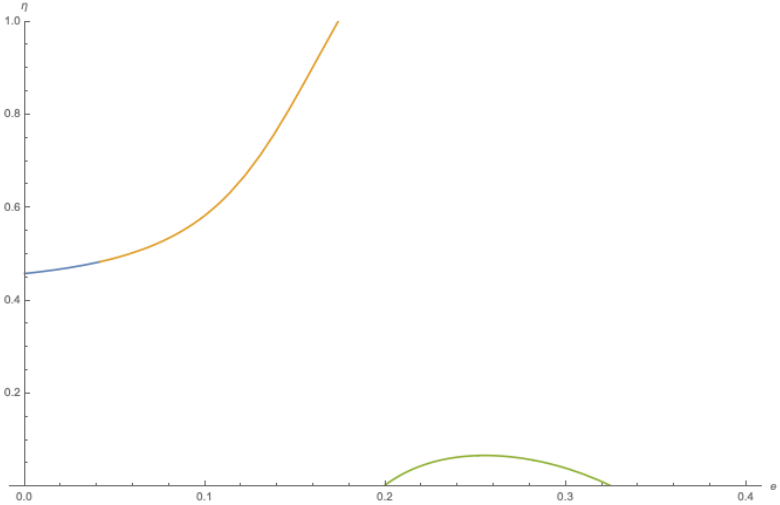
How does this change affect the model?

Bifurcation Diagram for e



Green branch is stable, the other two are unstable.

Note: to make this diagram, we used a simplification of the insolation function

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- Look at other spin-orbit resonances (relate it to [6])
- Recreate pictures from [6] using the Budyko Model
- Consider a dynamic albedo equation

Thank You!

Credits

[1] J. Checlair, K. Menou, and D. Abbot. *No Snowball on Habitable Tidally Locked Planets*, The Astrophys. J., 845 (2017), pp. 132 - 142.
[2] A.R. Dobrovolskis. *Insolation Patterns on Eccentric Exoplanets*, Icarus, 250 (2015), pp. 395 - 399.
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