Set 4 Solutions

1. Use the value of 0.0112372 for the ratio of carbon 13 to carbon 12 ($^{13}\text{C}:^{12}\text{C}$) in a standard PDB sample.
   
   a. Assume that the $^{13}\text{C}:^{12}\text{C}$ ratio in a calcium carbonate shell is 0.011. What is the value of $\delta^{13}\text{C}$, using PDB as a standard?

   **Solution.**
   
   \[ \delta^{13}\text{C} = \frac{0.011}{0.0112372} - 1 = -0.021108 \text{, or } -21.1\% . \]

   b. Carbon isotope analysis of a soil sample yields a value $\delta^{13}\text{C}$ at -17‰. What is the $^{13}\text{C}:^{12}\text{C}$ ratio in this sample?

   **Solution.**
   
   \[ r = 0.0112372(1 - 0.017) = 0.11046 \]

2. Suppose that the Earth were to fall into another ice age and that 3% of the current ocean were to evaporate and become glaciers with value of $\delta D$ at -450‰. Assuming that the current ocean has a VSMOW value of $\delta D$ at 0, what would be the value of $\delta D$ for the ice age ocean?

   **Solution.**
   
   Let $\delta_0 = 0$ be the value of $\delta D$ for the current ocean, let $\delta_1 = -0.45$ be the value of $\delta D$ for the new glaciers, and let $\delta_2$ be the value of $\delta D$ for the new ice age ocean. The formula derived in class relates these values:
   
   \[ \delta_0 \approx p\delta_1 + q\delta_2 , \]

   where $p = 0.03$ is the proportion of the current ocean that becomes glaciers and $q = 1 - p = 0.97$. Solving for $\delta_2$, we have
   
   \[ \delta_2 = \frac{\delta_0 - p\delta_1}{q} = \frac{0 - 0.03 \cdot (-0.45)}{0.97} = 0.0139 . \]

   So the new ocean would have a deuterium content of $\delta D = 13.9\%$.

3. Sea water evaporates to form water vapor, which condenses into clouds which rain into the ocean. The remaining water vapor is transported over land, where it forms new clouds, which rain into a lake. Finally, the remaining water vapor is transported over mountains, where it falls as snow.

   a. Assume that, when the water evaporates, the value of $\delta^{18}\text{O}$ in the sea water is 0 when the VSMOW standard is used. Assume also that the fractionation of oxygen 18 when the water evaporates from the sea is -10‰. What is the value of $\delta^{18}\text{O}$ in the water vapor?

   **Solution.** For a small amount of vapor at equilibrium, we have $\delta_1 = 0 - \varepsilon = -10\%$.

   Note that here and below we use the convention that positive epsilon means heavier water than vapor.
b. Assume that half of the water vapor condenses and rains back into the ocean, and assume that the fractionation of oxygen 18 when the water condenses is -11‰. What is the value of $\delta^{18}O$ in the rain falling into the ocean? What is the value of $\delta^{18}O$ in the vapor remaining in the atmosphere?

Solution. We use the formulae $\delta_1 = \delta_0 - \varepsilon q$ and $\delta_2 = \delta_0 + \varepsilon p$, where $\delta_0 = -10$‰ (from part (a)), $\varepsilon = 11$‰, and $p = q = 0.5$ to get $\delta_1 = -10 - 5.5 = -15.5$ and $\delta_2 = -10 + 5.5 = -4.5$. Therefore, the value of $\delta^{18}O$ in the rain is $-4.5$‰, while the value in the remaining vapor is $-15.5$‰.

c. Assume that half of the remaining water vapor condenses and rains into the lake, and assume that the fractionation of oxygen 18 under these conditions is -11.4‰. What is the value of $\delta^{18}O$ in the rain falling into the lake? What is the value of $\delta^{18}O$ in the vapor remaining in the atmosphere?

Solution. We use the same formulae with $\delta_0 = -15.5$‰ (from part (b)), $\varepsilon = 11.4$‰, and $p = q = 0.5$ to get $\delta_1 = -15.5 - 5.7 = -21.2$ and $\delta_2 = -15.5 + 5.7 = -9.8$. Therefore, the value of $\delta^{18}O$ in the rain is $-9.8$‰, while the value in the remaining vapor is $-21.2$‰.

d. Assume that half of the remaining water vapor condenses and falls as snow, and assume that the fractionation of oxygen 18 under these conditions is -11.7‰. What is the value of $\delta^{18}O$ in the snow?

Solution. We again use the same formulae with $\delta_0 = -21.2$‰ (from part (b)), $\varepsilon = 11.7$‰, and $p = q = 0.5$ to get $\delta_1 = -21.2 - 5.85 = -27.05$ and $\delta_2 = -21.2 + 5.85 = -15.35$. Therefore, the value of $\delta^{18}O$ in the snow is $-15.35$‰.