Show your work clearly; answers without explanation will receive no credit.

1. (3 Points) Substitute \( y = e^{rx} \) into the differential equation \( y'' + 5y' + 6y = 0 \) to determine all values of the constant \( r \) for which \( y = e^{rx} \) is a solution of the equation.

\[
(e^{rx})'' + 5(e^{rx})' + 6(e^{rx}) = 0
\]

\[
r^2 e^{rx} + 5re^{rx} + 6e^{rx} = 0
\]

\( e^{rx} \) is never zero, so we cancel this factor without losing solution

\[
r^2 + 5r + 6 = 0
\]

\[
(r + 2)(r + 3) = 0
\]

\[
r = -2 \text{ or } r = -3
\]

2. (4 Points) Find the position function \( x(t) \) of a moving particle with acceleration \( a(t) = 6t \), initial velocity \( v(0) = 12 \), and initial position \( x(0) = -1 \).

\[
a(t) = 6t
\]

\[
v(t) = \int a(t) \, dt + C = 3t^2 + C
\]

\[
v(0) = 12 \rightarrow C = 12
\]

\[
v(t) = 3t^2 + 12
\]

\[
x(t) = \int v(t) \, dt + \hat{C} = t^3 + 12t + \hat{C}
\]

\[
x(0) = -1 \rightarrow \hat{C} = -1
\]

\[
x(t) = t^3 + 12t - 1
\]

--- OVER for Problems 3 and 4 ---
3. (3 Points) Circle the statements that are guaranteed to be true about an initial value problem of the form
\[
\frac{dy}{dx} = y^2 \cdot h(x), \quad y(0) = 0
\]
where \( h \) is a continuous function of \( x \). Note: if \( f(x, y) = y^2 \cdot h(x) \), then \( \frac{\partial f}{\partial y} = 2y \cdot h(x) \).

\[\begin{array}{ll}
\text{A.} & \text{At least one solution exists, locally.} \quad F(x, y) = y^2 \cdot h(x) \text{ is continuous} \\
\text{B.} & \text{A unique solution exists, locally.} \quad \frac{\partial F}{\partial y} = 2y \cdot h(x) \text{ is continuous} \\
\text{X} & \text{There exists a global solution } y(x) \text{ (defined for all } x). \quad \frac{\partial F}{\partial y} = 2y \cdot h(x) \text{ is not bounded e.g. for } h(x) = 1, \text{ so by our usual test we can't conclude a global solution exists.}
\end{array}\]

However, \( y(x) = 0 \) is a global solution to the IVP, which goes to show our boundedness test on \( \frac{\partial F}{\partial y} \) is a sufficient but not necessary criterion for a global solution.

4. (2 points) Use separation of variables to find a general, implicit solution to the differential equation
\[
\frac{dy}{dx} = 2x \sec y
\]

\[
\frac{dy}{dx} = 2x \frac{1}{\cos y}
\]

\[\begin{align*}
\cos y \ dy &= 2x \ dx \\
\int \cos y \ dy &= \int 2x \ dx + C \\
\sin y &= x^2 + C
\end{align*}\]