1. Determine whether the vectors \((1, 0, 0), (3, -2, 0),\) and \((4, 9, 2)\) are linearly independent or linearly dependent, and explain your reasoning.

\[
\det \begin{bmatrix} 1 & 3 & 4 \\ 0 & -2 & 9 \\ 0 & 0 & 2 \end{bmatrix} = (1)(-2)(2) = -4 \neq 0 \text{ so the vectors are linearly independent}
\]

2. Find coefficients \(c_1\) and \(c_2\) so that \(c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 26 \\ 18 \end{bmatrix}\)

\[
\begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 26 \\ 18 \end{bmatrix} \quad \text{augmented matrix} \quad \begin{bmatrix} 2 & 6 & 26 \\ 1 & 4 & 18 \end{bmatrix} \quad \text{re-interpret as equations}
\]

\[
\begin{bmatrix} 1 & 3 & 13 \\ 0 & 1 & 5 \end{bmatrix} - 3R_2 \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 5 \end{bmatrix} \quad \text{re-interpreted as equations}
\]

\[
\begin{bmatrix} 1 & 3 & 13 \\ 0 & 1 & 5 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}
\]

3. Which of the following statements are true about the vectors in problem 2?

(A) \(\begin{bmatrix} 26 \\ 18 \end{bmatrix}\) can be expressed as a linear combination of \(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\) and \(\begin{bmatrix} 6 \\ 4 \end{bmatrix}\) True, that's the definition!

(B) \(\begin{bmatrix} 26 \\ 18 \\ 2 \\ 1 \\ 6 \\ 4 \end{bmatrix}\) are linearly dependent True, because \((-2)\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 5\begin{bmatrix} 6 \\ 4 \end{bmatrix} + (-1)\begin{bmatrix} 26 \\ 18 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\) (Also because they are three vectors in a 2D space)
4. Which of the following are subspaces of $\mathbb{R}^3$? Explain briefly.

(A) Any line in $\mathbb{R}^3$ that passes through the origin
   
   Subspace. Closed under addition and scalar multiplication

(B) Any plane in $\mathbb{R}^3$
   
   Not a subspace. Not closed under scalar multiplication unless the plane goes through $(0,0,0)$, since $0$ times any point (vector) in the plane is $(0,0,0)$.

(C) The region (octant) of $\mathbb{R}^3$ where all coordinate values are positive, i.e. $\{(x,y,z) : x > 0, y > 0, z > 0\}$
   
   Not a subspace, since not closed under scalar multiplication by a negative number.

Extra question to ponder: One of your homework problems asked you to show that the intersection of any two subspaces is a subspace. Is the union of any two subspaces a subspace? As an example, you might consider $U=$the $x$-axis and $V=$the $y$-axis as subspaces of $\mathbb{R}^2$.

No; $(1,0)$ is an element of $U$ and $(0,1)$ is an element of $V$, but their sum $(1,1)$ is not an elt of $U \cup V$.

And hence of $U \cup V$

Since $U \cup V$ is not closed under addition, it is not a subspace of $\mathbb{R}^2$.

This single counterexample in $\mathbb{R}^2$ proves that in general, the union of two subspaces is not necessarily a subspace.