No notes, texts, cellular devices or computers are allowed on this quiz. You may use a scientific calculator. Please show all work to receive full credit.

1. Let \( h(x) = 5x^2 - 3x^5 \).
   (a) Find the \( x \) values where \( h'(x) = 0 \)
   \[ h'(x) = 10x - 15x^4 = 5x(2 - 3x^3) = 0 \]
   \( x = 0 \) or \( 2 - 3x^3 = 0 \)
   \[ 3x^3 = 2 \]
   \[ x = \frac{2}{3} \]
   increasing on \( (0, \frac{2}{3}) \)
   decreasing on \( (-\infty, 0) \cup (\frac{2}{3}, \infty) \)

   (b) On what intervals is \( h \) increasing?
   (c) On what intervals is \( h \) decreasing?

   (d) Determine the local maxima and minima for \( h \)
   \[ \text{local min: (0, 0)} \quad \text{local max: } (\frac{2}{3}, 5\left(\frac{2}{3}\right)^2 - 3\left(\frac{2}{3}\right)^5) \]

   (e) Find the \( x \) values where \( h''(x) = 0 \)
   \[ h''(x) = 10 - 60x^2 = 0 \]
   \( x^2 = \frac{1}{6} \)
   \[ x = \frac{1}{\sqrt{6}} \]

   (f) On what intervals is \( h \) concave up?
   (g) On what intervals is \( h \) concave down?

   (h) What are the inflection points for \( h \)?
   \[ (\frac{2}{3}, 5\left(\frac{2}{3}\right)^2 - 3\left(\frac{2}{3}\right)^5) \]

2. Given that \( f(x) = \ln x \) is both continuous on the interval \( [1, 4] \) and differentiable on \( (1, 4) \), find all numbers \( c \) that satisfy the conclusion of the Mean Value Theorem.
   The average slope of \( \ln(x) \) on \( [1, 4] \) is
   \[ \frac{\ln(4) - \ln(1)}{4 - 1} = \frac{\ln(4)}{3} = \frac{\ln(4)}{3} \]
   The derivative of \( f(x) \) is \( f'(x) = \frac{1}{x} \)
   The Mean Value Theorem states that there exists \( c \) in \( (1, 4) \)
   with \( f'(c) \) = average slope of \( \ln(x) \) on \( [1, 4] \) = \( \frac{\ln(4)}{3} \)
   So we want \( \frac{1}{c} = \frac{\ln(4)}{3} \) so \( c = \frac{3}{\ln(4)} \)
   Check: is \( \frac{3}{\ln(4)} \) in \( (1, 4) \)? \( \ln(4) \approx 1.4 \), so \( \frac{3}{\ln(4)} \approx 2.2 \). Good!