No notes, texts, cellular devices or computers are allowed on this quiz. You may use a scientific calculator. Please show all work in order to receive full credit.

1. Use Newton's method with the initial approximation \( x_1 = -1 \) to find \( x_2 \) and \( x_3 \) (the second and third approximation to the root) of

\[
f(x) = x^7 + 4 = 0.
\]

\[
f'(x) = 7x^6.
\]

\[
x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -1 - \frac{(-1)^7 + 4}{7(-1)^6} = \frac{-10}{7}
\]

\[
x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \frac{-10}{7} - \frac{(-\frac{10}{7})^7 + 4}{7(-\frac{10}{7})^6} \approx -1.29
\]

2. Choose one of the following 2 word problems and circle the one that you want graded. (If you do not choose the problem to be graded, I will, and you may not like my choice!)

(a) A box with a square base and open top must have a volume of 32,000 cm\(^3\). Find the dimensions of the box that minimize the amount of material used.

\[
V = x^2y = 32,000 \quad \rightarrow \quad y = \frac{32,000}{x^2}
\]

Material used = \[
M = 4xy + 2x^2 = 4x \left( \frac{32,000}{x^2} \right) + x^2
\]

So, want to minimize \( M(x) = \frac{128,000}{x} + x^2 \)

on interval \([0, \infty)\)

(x must be positive but can get as large as we'd like provided we make \(y\) small to compensate) (likewise can get close to zero if we make \(y\) large)

Candidates for minimum: \( M'(x) = 0 \), \( M'(x) \text{ DNE} \), and endpoints

(b) Find the point on the curve \( y = \sqrt{x} \) that is closest to the point \((0, 3)\).

\[
Z = \sqrt{(0-x)^2 + (3-y)^2} = \sqrt{(0-x)^2 + (3-\sqrt{x})^2}
\]

\[
Z(x) = \sqrt{x^2 + (3-\sqrt{x})^2}
\]

Interval of \(x\) values to consider: \([0, \infty)\)

Candidates for absolute minimum:

where \(Z'(x) = 0\), \(Z'(x) \text{ DNE} \), and end point

(next page)
2) continued

\[ M'(x) = \frac{-128,000}{x^2} + 2x = 0 \]

\[
-128,000 + 2x^3 = 0 \\
x^3 = 64,000 \\
x = 40
\]

\[ M'(x) \text{ DNE when } x = 0 \text{ but that's not in our interval} \]

endpoints: n/a

So minimum must occur at \[ x = 40 \, \text{cm} \Rightarrow y = \frac{32,000}{x^2} = \frac{y^2}{120 \, \text{cm}} \]

A good check: Use first or second derivative test to confirm this gives a local minimum

2) continued

\[ Z'(x) = \left( \frac{1}{2\sqrt{x^3 + (3-x\sqrt{x})^2}} \right) \left( 2x + 2(3-x\sqrt{x}) \left( \frac{-1}{2\sqrt{x}} \right) \right) \]  

(Lots of chain rule!)

\[ Z'(x) = 0 \text{ when } 2x + \frac{Z(3-x\sqrt{x})}{2\sqrt{x}} = 0 \]

\[ 2x + (\text{other terms}) = 0 \]

\[ 2x + 1 - \frac{3}{\sqrt{x}} = 0 \]

\[ 2x^{3/2} + x^{1/2} - 3 = 0 \quad [\text{hard to solve}] \]

\[ Z'(x) \text{ does not exist when } x = 0 \]

endpoint: \[ x = 0 \]

Compare \[ Z(0) = 3 \]

\[ Z(\text{solution to } Z'(x)=0) = ? \quad \text{full credit} \]

\[ \text{for getting to here} \]